Global Output Feedback Tracking Control for Uncertain Second-Order Nonlinear Systems

H. T. Dinh, S. Bhasin, D. Kim, and W. E. Dixon

***Abstract*—A dynamic neural network (DNN) observer-based output feedback controller for uncertain nonlinear systems with bounded disturbances is developed. The DNN-based observer works in conjunction with a dynamic filter for state estimation using only output measurements during on-line operation. A sliding mode term is included in the DNN structure to robustly account for exogenous disturbances and reconstruction errors. Weight update laws for the DNN, based on estimation errors, tracking errors, and the filter output are developed which guarantee global asymptotic regulation of the state estimation error. A combination of a DNN feedforward term, along with estimated state feedback and sliding mode terms yield a global asymptotic tracking result. The developed method yields the first output feedback technique simultaneously achieving global asymptotic tracking and global asymptotic estimation of unmeasurable states for the class of uncertain nonlinear systems with bounded disturbances. A two-link robot manipulator is used to investigate the performance of the proposed control approach.**

I. Introduction

The problem of output feedback (OFB) tracking control for nonlinear dynamic systems has been a topic of considerable interest over the past several decades. Motivation arises from the fact that full access to system states is sometimes impossible in many practical systems. An obvious method to estimate the unmeasurable states is using *ad hoc* numerical differentiation. The simplicity of this technique makes it particularly useful for implementation. However, if output measurements are noisy, such numerical techniques will amplify the high frequency content which may produce undesired oscillations or even system instability. Other solutions can be classified as observer-based or filter-based techniques that utilize the output information for estimating unmeasurable states. While observers estimate the output derivative by approximating the system dynamics, filters approximate the behavior of a differentiator over a range of frequencies. Hence, observer designs need partial or exact model knowledge of the system dynamics, whereas filters can provide a model-free means of estimating unmeasurable states.

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OFB controllers using model-based observers were developed in [1]–[3], based on the assumption of exact model knowledge. OFB control for systems with parametric uncertainties have been developed in [4]–[6]. However, a limitation of such previous adaptive OFB control approaches is that only linear-in-the-parameters (LP) uncertainties are considered. As a result, if uncertainties in the system do not satisfy the LP condition or if the system is affected by disturbances, the results will reduce to a uniformly ultimately bounded result.

The condition of linear dependence upon unknown parameters can be relaxed by introducing a neural network (NN) or fuzzy logic in the observer structure as in [7]–[12]; however, both estimation and tracking errors are only guaranteed to be bounded due to the existence of reconstruction errors. The first semi-global asymptotic OFB tracking result for second-order dynamic systems under the condition that uncertain dynamics are first-order differentiable was introduced in [13] with a novel filter design. All of the uncertain nonlinearities in [13] are damped out by a sliding mode term, so the discontinuous controller requires high-gain. However, it is not clear how to simply add a NN-based feedforward estimation of the nonlinearities in results such as [13] to mitigate the high-gain condition, because of the need to inject nonlinear functions of the unmeasurable state. The approach used in this paper avoids this issue by exploiting the recurrent nature of a dynamic neural network (DNN) structure to inject terms that cancel cross terms associated with the unmeasurable state.

In this paper, and the preliminary work in [14], a DNN-based observer-controller is proposed for uncertain nonlinear systems affected by bounded disturbances, to achieve a two-fold result: asymptotic estimation of the unmeasurable states and asymptotic tracking control. The uncertain dynamics are assumed to be first-order differentiable. The universal approximation property of DNNs is utilized to approximate the uncertain nonlinear system. A modified version of the filter introduced in [13] is used to estimate the output derivative. A combination of a NN feedforward term, along with estimated state feedback and sliding mode terms are designed for the controller. The DNN observer adapts on-line for nonlinear uncertainties and should heuristically perform better than a robust feedback observer. Weight update laws for the DNN based on the estimation error, tracking error, and filter output are proposed. Global asymptotic regulation of the estimation error and global asymptotic tracking are achieved. Experiments on a two-link robot manipulator show the effectiveness of the developed method compared with a PID controller and the approach in [13].

II. System Model and Objectives

Consider a control-affine second order Euler-Lagrange-like nonlinear system of the form

$$\ddot{x} = f(x, \dot{x}) + G(x)u + d, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the measurable output with a finite initial condition $x(0) = x_0$, $u(t) \in \mathbb{R}^n$ is the control input, $f(x, \dot{x}) \in \mathbb{R}^n, G(x) \in \mathbb{R}^{n \times n}$ are continuous functions, and $d(t) \in \mathbb{R}^n$ is an exogenous disturbance. The following assumptions about the system in (1) will be utilized in the subsequent development.

Assumption 1. The time derivatives of the system output $\dot{x}(t), \ddot{x}(t)$ are unmeasurable. **Assumption 2.** The unknown function $f(x, \dot{x})$ is C^1, and the function $G(x)$ is known, invertible and the matrix inverse $G^{-1}(x)$ is bounded. **Assumption 3.** The disturbance $d(t)$ is differentiable, and $d(t), \dot{d}(t) \in \mathcal{L}_\infty$.

The universal approximation property of multilayer NNs (MLNN) states that given any continuous function $F : \mathbb{S} \to \mathbb{R}^n$, where $\mathbb{S}$ is a compact set, there exist ideal weights $\theta = \theta^*$, such that the output of the NN, $\hat{F}(\cdot, \theta)$ approximates $F(\cdot)$ to an arbitrary accuracy [15], [16]. Hence, the unknown function $f(x, \dot{x})$ in (1) can be replaced by a multi-layer NN (MLNN), and the system can be represented as

$$\ddot{x} = W^T \sigma(V_1^T x + V_2^T \dot{x}) + \varepsilon(x, \dot{x}) + Gu + d, \tag{2}$$

where $W \in \mathbb{R}^{N+1 \times n}$, $V_1, V_2 \in \mathbb{R}^{n \times N}$ are unknown ideal weight matrices of the MLNN having N hidden layer neurons, $\sigma(t) \triangleq \sigma(V_1^T x(t) + V_2^T \dot{x}(t)) \in \mathbb{R}^{N+1}$ is the activation function (sigmoid, hyperbolic tangent etc.), and $\varepsilon(x, \dot{x}) \in \mathbb{R}^n$ is a function reconstruction error. The following assumptions will be used in the DNN-based observer and controller development and stability analysis.

Assumption 4. The ideal NN weights are bounded by known positive constants [17], i.e. $\|W\| \leq \bar{W}$, $\|V_1\| \leq \bar{V}_1$, $\|V_2\| \leq \bar{V}_2$. **Assumption 5.** The activation function $\sigma(\cdot)$ and its partial derivatives $\sigma'(\cdot)$, $\sigma''(\cdot)$ are bounded [17]. This assumption is satisfied for typical activation functions (e.g., sigmoid, hyperbolic tangent). **Assumption 6.** The function reconstruction error $\varepsilon(x, \dot{x})$, and its first time derivative are bounded [17], as $\|\varepsilon(x, \dot{x})\| \leq \bar{\varepsilon}_1$, $\|\dot{\varepsilon}(x, \dot{x}, \ddot{x})\| \leq \bar{\varepsilon}_2$, where $\bar{\varepsilon}_1, \bar{\varepsilon}_2$ are known positive constants.

A contribution of this paper is the development of a robust DNN-based observer such that the estimated states globally asymptotically converge to the real states of the system (1), and a discontinuous controller enables the system state to globally asymptotically track a desired time-varying trajectory $x_d(t) \in \mathbb{R}^n$, despite uncertainties and disturbances in the system. To quantify these objectives, an estimation error $\tilde{x}(t) \in \mathbb{R}^n$ and a tracking error $e(t) \in \mathbb{R}^n$ are defined as

$$\tilde{x} \triangleq x - \hat{x}, \quad e \triangleq x - x_d, \tag{3}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the state of the DNN observer which is introduced in the subsequent development. The desired trajectory $x_d(t)$ and its derivatives $x_d^{(i)}(t)$ $(i = 1, 2)$, are assumed to exist and be bounded. To compensate for the lack of direct measurements of $\dot{x}(t)$, a filtered estimation error, $r_{es}(t) \in \mathbb{R}^n$, and a filtered tracking error, $r_{tr}(t) \in \mathbb{R}^n$, are defined as

$$r_{es} \triangleq \dot{\tilde{x}} + \alpha\tilde{x} + \eta, \quad r_{tr} \triangleq \dot{e} + \alpha e + \eta, \tag{4}$$

where $\alpha \in \mathbb{R}$ is a positive constant gain, and $\eta(t) \in \mathbb{R}^n$ is an output of the dynamic filter

$$\begin{aligned} &\eta = p - (k + \alpha)\tilde{x}, \\ &\dot{p} = -(k + 2\alpha)p - \nu + ((k + \alpha)^2 + 1)\tilde{x} + e, \\ &\dot{\nu} = p - \alpha\nu - (k + \alpha)\tilde{x}, \; p(0) = (k + \alpha)\tilde{x}(0), \; \nu(0) = 0, \end{aligned} \tag{5}$$

where $\nu(t) \in \mathbb{R}^n$ is another output of the filter, $p(t) \in \mathbb{R}^n$ is used as an internal filter variable, and $k \in \mathbb{R}$ is a positive constant control gain. The filtered estimation error $r_{es}(t)$ and the filtered tracking error $r_{tr}(t)$ are not measurable since the expressions in (4) depend on $\dot{x}(t)$.

Remark 1. The basic structure of the second order dynamic filter in (5) was first proposed in [13]. The filter in (5) admits the estimation error $\tilde{x}(t)$ and the tracking error $e(t)$ as its inputs and produces two signal outputs $\nu(t)$ and $\eta(t)$. An interesting point is that there is a virtual filter inside the introduced filter where $\eta(t)$ is the filtered signal of $\nu(t)$ since $\eta(t)$, $\nu(t)$ are related as $\eta = \dot{\nu} + \alpha\nu$. The auxiliary signal $p(t)$ is utilized to only generate the signal $\eta(t)$ without involving the unmeasurable state $\dot{x}(t)$. Hence, the filter can be physically implemented since it depends only on the estimation error $\tilde{x}(t)$ and the tracking error $e(t)$ which are measurable.

III. DNN-based Robust Observer

The following multi-layer dynamic neural network (MLDNN) architecture is proposed to observe the system in (1)

$$\ddot{\hat{x}} = \hat{W}^T \hat{\sigma} + Gu - (k + 3\alpha)\eta + \beta_1 sgn(\tilde{x} + \nu), \tag{6}$$

where $\begin{bmatrix} \hat{x}(t)^T & \dot{\hat{x}}(t)^T \end{bmatrix}^T \in \mathbb{R}^{2n}$ are the states of the DNN observer, $\hat{W}(t) \in \mathbb{R}^{N+1 \times n}$, $\hat{V}_1(t), \hat{V}_2(t) \in \mathbb{R}^{n \times N}$ are the weight estimates, $\hat{\sigma}(t) \triangleq \sigma(\hat{V}_1(t)^T \hat{x}(t) + \hat{V}_2(t)^T \dot{\hat{x}}(t)) \in \mathbb{R}^{N+1}$ and $\beta_1 \in \mathbb{R}$ is a positive constant control gain.

Remark 2. The term $(k + 3\alpha)\eta(t)$ in the DNN observer in (6) is a cross-term which is cancelled in the stability analysis. The sliding mode term $sgn(\tilde{x}(t) + \nu(t))$ is added to the observer structure to provide robustness against NN reconstruction errors and external disturbances. The NN term $\hat{W}(t)^T \hat{\sigma}(t)$ receives feedback of the observer states $\hat{x}(t), \dot{\hat{x}}(t)$ as inputs; hence the observer exploits a DNN structure. Motivation for the DNN-based observer design is that the DNN is proven to approximate nonlinear dynamic systems with any degree of accuracy [15], [18], and the DNN includes state feedback yielding computational advantages over a static feedforward NN [19].

The weight update laws for the DNN in (6) are developed based on the subsequent stability analysis as

$$\begin{aligned}\dot{\hat{W}} &= \Gamma_w proj[\hat{\sigma}_d(\tilde{x}+e+2\nu)^T],\\ \dot{\hat{V}}_1 &= \Gamma_{v1} proj[x_d(\tilde{x}+e+2\nu)^T\hat{W}^T\hat{\sigma}'_d],\\ \dot{\hat{V}}_2 &= \Gamma_{v2} proj[\dot{x}_d(\tilde{x}+e+2\nu)^T\hat{W}^T\hat{\sigma}'_d],\end{aligned} \quad (7)$$

where $\Gamma_w \in \mathbb{R}^{(N+1)\times(N+1)}$, $\Gamma_{v1}, \Gamma_{v2} \in \mathbb{R}^{n\times n}$, are constant symmetric positive-definite adaptation gains, the terms $\hat{\sigma}_d(t), \hat{\sigma}'_d(t)$ are defined as $\hat{\sigma}_d(t) \triangleq \sigma(\hat{V}_1(t)^T x_d(t) + \hat{V}_2(t)^T \dot{x}_d(t))$, $\hat{\sigma}'_d(t) \triangleq d\sigma(\varsigma)/d\varsigma|_{\varsigma=\hat{V}_1^T x_d+\hat{V}_2^T\dot{x}_d}$, and $proj(\cdot)$ is a smooth projection operator [20], [21] used to guarantee that the weight estimates $\hat{W}(t), \hat{V}_1(t), \hat{V}_2(t)$ remain bounded.

To facilitate the subsequent analysis, (4) and (5) can be used to express the time derivative of $\eta(t)$ as

$$\dot{\eta} = -(k+\alpha)r_{es} - \alpha\eta + \tilde{x} + e - \nu. \quad (8)$$

The closed-loop dynamics of the filtered estimation error in (4) can be determined by using (2)-(4), (6) and (8) as

$$\begin{aligned}\dot{r}_{es} &= W^T\sigma - \hat{W}^T\hat{\sigma} + \varepsilon + d + (k+3\alpha)\eta - \beta_1 sgn(\tilde{x}+\nu)\\ &\quad + \alpha(r_{es} - \alpha\tilde{x} - \eta) - (k+\alpha)r_{es} - \alpha\eta + \tilde{x} + e - \nu.\end{aligned} \quad (9)$$

Adding and subtracting $W^T\sigma_d + W^T\hat{\sigma}_d + \hat{W}^T\hat{\sigma}_d$, where $\sigma_d(t) \triangleq \sigma(V_1^T x_d(t) + V_2^T \dot{x}_d(t))$, the expression in (9) can be rewritten as

$$\dot{r}_{es} = \tilde{N}_1 + N - kr_{es} - \beta_1 sgn(\tilde{x}+\nu) + (k+\alpha)\eta - \tilde{x}, \quad (10)$$

where the auxiliary function $\tilde{N}_1(e, \tilde{x}, \nu, r_{es}, r_{tr}, \hat{W}, \hat{V}_1, \hat{V}_2, t) \in \mathbb{R}^n$ is defined as

$$\tilde{N}_1 \triangleq W^T(\sigma - \sigma_d) - \hat{W}^T(\hat{\sigma} - \hat{\sigma}_d) - (\alpha^2 - 2)\tilde{x} - \nu + e, \quad (11)$$

and $N(x_d, \dot{x}_d, \hat{W}, \hat{V}_1, \hat{V}_2, t) \in \mathbb{R}^n$ is segregated into two parts as

$$N \triangleq N_D + N_B. \quad (12)$$

In (12), $N_D(t)$, $N_B(\hat{W}, \hat{V}_1, \hat{V}_2, t) \in \mathbb{R}^n$ are defined as

$$N_D \triangleq \varepsilon + d, \quad N_B \triangleq N_{B_1} + N_{B_2}. \quad (13)$$

In (13), $N_{B_1}(\hat{W}, \hat{V}_1, \hat{V}_2, t)$, $N_{B_2}(\hat{W}, \hat{V}_1, \hat{V}_2, t) \in \mathbb{R}^n$ are defined as

$$\begin{aligned}N_{B_1} &\triangleq W^T O(\tilde{V}_1^T x_d + \tilde{V}_2^T \dot{x}_d)^2 + \tilde{W}^T\hat{\sigma}'_d(\tilde{V}_1^T x_d + \tilde{V}_2^T\dot{x}_d),\\ N_{B_2} &\triangleq \tilde{W}^T\hat{\sigma}_d + \hat{W}^T\hat{\sigma}'_d(\tilde{V}_1^T x_d + \tilde{V}_2^T\dot{x}_d),\end{aligned} \quad (14)$$

where $\tilde{W}(t) \triangleq W - \hat{W}(t) \in \mathbb{R}^{N+1\times n}$, $\tilde{V}_1(t) \triangleq V_1 - \hat{V}_1(t) \in \mathbb{R}^{n\times N}$, $\tilde{V}_2(t) \triangleq V_2 - \hat{V}_2(t) \in \mathbb{R}^{n\times N}$ are the estimate mismatches for the ideal NN weights, and $O(\tilde{V}_1^T x_d + \tilde{V}_2^T\dot{x}_d)^2 \in \mathbb{R}^{N+1}$ is the higher order term in the Taylor series of the vector functions $\sigma_d(\cdot)$ in the neighborhood of $\hat{V}_1^T x_d + \hat{V}_2^T\dot{x}_d$ as

$$\sigma_d = \hat{\sigma}_d + \hat{\sigma}'_d(\tilde{V}_1^T x_d + \tilde{V}_2^T\dot{x}_d) + O(\tilde{V}_1^T x_d + \tilde{V}_2^T\dot{x}_d)^2. \quad (15)$$

Motivation for segregating the terms in (10), (12) and (13) is derived from the fact that different terms have different bounds. The term $\tilde{N}_1(\cdot)$ includes all terms which can be upper bounded by states, whereas $N(\cdot)$ includes all terms which can be upper bounded by constants. The difference between the terms $N_D(\cdot)$ and $N_B(\cdot)$ in (12) is that the first time derivative of $N_D(\cdot)$ is upper-bounded by a constant, whereas the term $\dot{N}_B(\cdot)$ is state dependent. The term $N_B(\cdot)$ is further segregated as (13) to aid in the weight update law design for the DNN in (7). In the subsequent stability analysis, the term $N_{B_1}(\cdot)$ is cancelled by the error feedback and the sliding mode term, while the term $N_{B_2}(\cdot)$ is partially compensated for by the weight update laws and partially cancelled by the error feedback and the sliding mode term.

Using (3), (4), Assumptions 4, 5, the $proj(\cdot)$ algorithm in (7) and the Mean Value Theorem [22], the auxiliary function $\tilde{N}_1(t)$ in (11) can be upper-bounded as

$$\left\|\tilde{N}_1\right\| \leq \zeta_1 \|z\|, \quad (16)$$

where $\zeta_1 \in \mathbb{R}$ is a computable positive constant, and $z(\tilde{x}, e, r_{es}, r_{tr}, \nu, \eta) \in \mathbb{R}^{6n}$ is defined as

$$z \triangleq [\tilde{x}^T \; e^T \; r_{es}^T \; r_{tr}^T \; \nu^T \; \eta^T]^T. \quad (17)$$

Based on Assumptions $3-6$, the Taylor series expansion in (15), and the weight update laws in (7), the following bounds can be developed

$$\begin{aligned}&\|N_D\| \leq \zeta_2, \;\; \|N_{B_1}\| \leq \zeta_3, \;\; \|N_{B_2}\| \leq \zeta_4,\\ &\left\|\dot{N}_D\right\| \leq \zeta_5, \;\; \left\|\dot{N}_B\right\| \leq \zeta_6 + \zeta_7\|z\|,\end{aligned} \quad (18)$$

where $\zeta_i \in \mathbb{R}, i = 2, 3, ..., 7$, are computable positive constants.

IV. Robust Adaptive Tracking Controller

The control objective is to force the system state to asymptotically track the desired trajectory $x_d(t)$, despite uncertainties and disturbances in the system. Quantitatively, the objective is to regulate the filtered tracking controller $r_{tr}(t)$ to zero. Using (2)-(4) and (8), the open-loop dynamics of the tracking error in (4) are expressed as

$$\begin{aligned}\dot{r}_{tr} &= W^T\sigma + Gu + \varepsilon + d - \ddot{x}_d + \alpha(r_{tr} - \alpha e - \eta)\\ &\quad - (k+\alpha)r_{es} - \alpha\eta + \tilde{x} + e - \nu.\end{aligned} \quad (19)$$

The control input $u(t)$ is now designed as a composition of the DNN term, the estimated states $\hat{x}(t), \dot{\hat{x}}(t)$, and the sliding mode term as

$$u(t) = G^{-1}[\ddot{x}_d - \hat{W}^T\hat{\sigma}_d - (k+\alpha)(\dot{\hat{e}} + \alpha\hat{e}) - \beta_2 sgn(e+\nu)], \quad (20)$$

where $\beta_2 \in \mathbb{R}$ is a positive constant control gain, and the tracking error estimate $\hat{e}(t) \in \mathbb{R}^n$ is defined as $\hat{e} \triangleq \hat{x} - x_d$. Based on the fact that the estimated states are measurable, the tracking error estimate $\hat{e}(t)$ and its derivative $\dot{\hat{e}}(t)$ are measurable; moreover, $r_{tr}(t)$ is related to $r_{es}(t)$ as

$$r_{tr} = r_{es} + \dot{\hat{e}} + \alpha\hat{e}. \quad (21)$$

Adding and subtracting $W^T\sigma_d + W^T\hat{\sigma}_d$ and using (19)-(21), the closed-loop error system becomes

$$\dot{r}_{tr} = \tilde{N}_2 + N - kr_{tr} - \beta_2 sgn(e+\nu) - e, \quad (22)$$

where the auxiliary function $\tilde{N}_2(e,\tilde{x},\eta,\nu,r_{tr},t)\in\mathbb{R}^n$ is defined as

$$\tilde{N}_2\triangleq W^T(\sigma-\sigma_d)-(\alpha^2-2)e-\nu+\tilde{x}-2\alpha\eta, \quad (23)$$

and the function $N(\cdot)$ is introduced in (12). Similarly, using (3), (4), Assumptions 4, 5, the $proj(\cdot)$ algorithm in (7), and Mean Value Theorem [22], the auxiliary function $\tilde{N}_2(\cdot)$ in (23) can be upper-bounded as

$$\left\|\tilde{N}_2\right\|\leq\zeta_8\left\|z\right\|, \quad (24)$$

where $\zeta_8\in\mathbb{R}$ is a computable positive constant.

To facilitate the subsequent stability analysis, let $y(z,P,Q)\in\mathbb{R}^{6n+2}$ be defined as $y\triangleq[z^T\ \ \sqrt{P}\ \ \sqrt{Q}]^T$, where the auxiliary function $P(\tilde{x},e,\nu,\dot{\tilde{x}},\dot{e},\dot{\nu},t)\in\mathbb{R}$ is the Filippov solution to the differential equation

$$\dot{P}\triangleq L, \quad (25)$$

$$P_0=\beta_1\sum_{j=1}^{n}\left|\tilde{x}_j(0)+\nu_j(0)\right|+\beta_2\sum_{j=1}^{n}\left|e_j(0)+\nu_j(0)\right| -\left(\tilde{x}(0)+e(0)+2\nu(0)\right)^T N\left(0\right),$$

where the subscript $j=1,2,..,n$ denotes the j^{th} element of $\tilde{x}(0)$, $e(0)$ or $\nu(0)$, and the auxiliary function $L(\tilde{x},e,\nu,\dot{\tilde{x}},\dot{e},\dot{\nu},t)\in\mathbb{R}$ is defined as

$$\begin{aligned}L\triangleq&-r_{es}^T(N_D+N_{B_1}-\beta_1 sgn(\tilde{x}+\nu))\\&-r_{tr}^T(N_D+N_{B_1}-\beta_2 sgn(e+\nu))\\&-(\dot{\tilde{x}}+\dot{e}+2\dot{\nu})^T N_{B_2}+\beta_3\left\|z\right\|^2,\end{aligned} \quad (26)$$

where β_1, β_2 are introduced in (6) and (20), and $\beta_3\in\mathbb{R}$ is a positive constant. The control gains β_i, $i=1,2,3$ are selected according to the sufficient conditions

$$\begin{aligned}&\beta_1,\beta_2>\max(\zeta_2+\zeta_3+\zeta_4,\zeta_2+\zeta_3+\frac{\zeta_5}{\alpha}+\frac{\zeta_6}{\alpha}),\\&\beta_3>2\zeta_7,\end{aligned} \quad (27)$$

where ζ_i, $i=1,2,...,7$ are introduced in (16) and (18). Provided the sufficient conditions in (27) are satisfied, the following inequality can be obtained $P(\cdot)\geq 0$ (see [23]). The auxiliary function $Q(\tilde{W},\tilde{V}_1,\tilde{V}_2)\in\mathbb{R}$ is defined as

$$Q\triangleq\frac{\alpha}{2}\left[tr(\tilde{W}^T\Gamma_w^{-1}\tilde{W})+tr(\tilde{V}_1^T\Gamma_{v1}^{-1}\tilde{V}_1)+tr(\tilde{V}_2^T\Gamma_{v2}^{-1}\tilde{V}_2)\right], \quad (28)$$

where $tr(\cdot)$ denotes the trace of a matrix. Since the gains $\Gamma_w,\Gamma_{v1},\Gamma_{v2}$ are symmetric, positive-definite matrices, $Q(\cdot)\geq 0$.

V. Lyapunov Stability Analysis For DNN-based Observation and Control

Theorem 1. *The DNN-based observer and controller proposed in (6) and (20), respectively, along with the weight update laws in (7) ensure global asymptotic estimation and tracking in sense that*

$$\left\|\dot{\tilde{x}}(t)\right\|\to 0\ as\ t\to\infty,\ and\ \left\|e(t)\right\|\to 0\ \ as\ t\to\infty,$$

provided the gain conditions in (27) are satisfied, and the control gains α and $k=k_1+k_2$ introduced in (4)-(5) are selected as

$$\lambda\triangleq\min(\alpha,k_1)>\frac{\zeta_1^2+\zeta_8^2}{4k_2}+\beta_3, \quad (29)$$

where ζ_1,ζ_8,β_3 are introduced in (16), (24), and (26), respectively.

Proof: Consider the Lyapunov candidate function $V_L(y,t):\mathcal{D}\times(0,\infty)\to\mathbb{R}$, which is a Lipschitz continuous positive definite function defined as

$$\begin{aligned}V_L\triangleq&\frac{1}{2}\tilde{x}^T\tilde{x}+\frac{1}{2}e^Te+\frac{1}{2}\nu^T\nu+\frac{1}{2}\eta^T\eta+\frac{1}{2}r_{es}^Tr_{es}\\&+\frac{1}{2}r_{tr}^Tr_{tr}+P+Q,\end{aligned} \quad (30)$$

which satisfies the following inequalities:

$$U_1(y)\leq V_L(y,t)\leq U_2(y). \quad (31)$$

In (31), $U_1(y)$, $U_2(y)\in\mathbb{R}$ are continuous positive definite functions defined as $U_1(y)\triangleq\frac{1}{2}\left\|y\right\|^2$, and $U_2(y)\triangleq\left\|y\right\|^2$.

Let $\dot{y}=h(y,t)$ represent the closed-loop differential equations in (4)-(7), (8), (10), (22), and (25), where $h(y,t)\in\mathbb{R}^{6n+2}$ denotes the right-hand side of the closed-loop error signals. Using Filippov's theory of differential inclusions [24]–[27], the existence of solutions can be established for $\dot{y}\in K[h](y,t)$, where $K[h]\triangleq\underset{\delta>0}{\cap}\underset{\mu M=0}{\cap}\overline{co}h(B(y,\delta)-M,t)$, where $\underset{\mu M=0}{\cap}$ denotes the intersection of all sets M of Lebesgue measure zero, $\overline{co}$ denotes convex closure, and $B(y,\delta)=\left\{w\in R^{6n+2}|\left\|y-w\right\|<\delta\right\}$. The right hand side of the differential equation, $h(y,t)$, is continuous except for the Lebesgue measure zero set of times $t\in[t_0,t_f]$ when $\tilde{x}(t)+\nu(t)=0$ or $e(t)+\nu(t)=0$. Hence, the set of time instances for which $\dot{y}(t)$ is not defined is Lebesgue negligible. The absolutely continuous solution $y(t)=y(t_0)+\int_{t_0}^{t}\dot{y}(t)dt$ does not depend on the value of $\dot{y}(t)$ on a Lebesgue negligible set of time-instances [28]. Under Filippov's framework, a generalized Lyapunov stability theory can be used (see [27], [29]–[31] for further details) to establish strong stability of the closed-loop system $\dot{y}=h(y,t)$. The generalized time derivative of (30) exists almost everywhere (a.e.), i.e. for almost all $t\in[t_0,t_f]$, and $\dot{V}_L(y)\in^{a.e.}\dot{\tilde{V}}_L(y)$, where $\dot{\tilde{V}}_L=\underset{\xi\in\partial V_L(y)}{\cap}\xi^TK\left[\Psi\right]^T$, ∂V_L is the generalized gradient of $V_L(y)$ [29], and $\Psi\triangleq\left[\dot{\tilde{x}}^T\ \dot{e}^T\ \dot{\nu}^T\ \dot{\eta}^T\ \dot{r}_{es}^T\ \dot{r}_{tr}^T\ \frac{1}{2}P^{-\frac{1}{2}}\dot{P}\ \frac{1}{2}Q^{-\frac{1}{2}}\dot{Q}\right]$. Since $V_L(y)$ is a locally Lipschitz continuous regular function that is smooth in y, $\dot{\tilde{V}}_L(y)$ can be simplified as [30]

$$\dot{\tilde{V}}_L=\nabla V^TK\Psi^T=\left[\tilde{x}^T\ e^T\ \nu^T\ \eta^T\ r_{es}^T\ r_{tr}^T\ 2P^{\frac{1}{2}}\ 2Q^{\frac{1}{2}}\right]K\Psi^T.$$

Using the calculus for $K\left[\cdot\right]$ from [31] (Theorem 1, Properties 2, 5, 7), and substituting the dynamics from (4), (5), (8), (10), (22), (25), (26) and (28), $\dot{\tilde{V}}_L(y)$ can be rewritten as

$$\begin{aligned}\dot{\tilde{V}}_L \subset\ & \tilde{x}^T(r_{es}-\alpha\tilde{x}-\eta)+e^T(r_{tr}-\alpha e-\eta)\\ &+\eta^T[-(k+\alpha)r_{es}-\alpha\eta+\tilde{x}+e-\nu]\\ &+\nu^T(\eta-\alpha\nu)+r_{es}^T\{(k+\alpha)\eta-\tilde{x}\}\\ &+r_{es}^T\{\tilde{N}_1+N-kr_{es}-\beta_1K[sgn(\tilde{x}+\nu)]\}\\ &+r_{tr}^T\{\tilde{N}_2+N-kr_{tr}-\beta_2K[sgn(e+\nu)]-e\}\\ &-r_{es}^T\{N_D+N_{B_1}-\beta_1K[sgn(\tilde{x}+\nu)]\}\\ &-r_{tr}^T\{N_D+N_{B_1}-\beta_2K[sgn(e+\nu)]\}+\beta_3\|z\|^2\\ &-(\dot{\tilde{x}}+\dot{e}+2\dot{\nu})^TN_{B_2}-\alpha tr(\tilde{W}^T\Gamma_w^{-1}\dot{\hat{W}})\\ &-\alpha tr(\tilde{V}_1^T\Gamma_{v1}^{-1}\dot{\hat{V}}_1)-\alpha tr(\tilde{V}_2^T\Gamma_{v2}^{-1}\dot{\hat{V}}_2). \end{aligned} \tag{32}$$

The set in (32) reduces to a scalar inequality, since the RHS is continuous except for the Lebesgue measure zero set of times when $\tilde{x}(t)+\nu(t)=0$ or $e(t)+\nu(t)=0$. Substituting the weight update laws in (7) and canceling common terms, (32) can be upper bounded as

$$\begin{aligned}\dot{\tilde{V}}_L \overset{a.e.}{\leq}\ & -\alpha\tilde{x}^T\tilde{x}-\alpha e^Te-\alpha\nu^T\nu-\alpha\eta^T\eta-kr_{es}^Tr_{es}\\ &-kr_{tr}^Tr_{tr}+r_{es}^T\tilde{N}_1+r_{tr}^T\tilde{N}_2+\beta_3\|z\|^2. \end{aligned} \tag{33}$$

Using (16) and (24), substituting $k=k_1+k_2$, and completing the squares, the expression in (33) can be further bounded as

$$\begin{aligned}\dot{\tilde{V}}_L \overset{a.e.}{\leq}\ & -\alpha\|\tilde{x}\|^2-\alpha\|e\|^2-\alpha\|\nu\|^2-\alpha\|\eta\|^2-k_1\|r_{es}\|^2\\ &-k_1\|r_{tr}\|^2+\left(\frac{\zeta_1^2+\zeta_8^2}{4k_2}+\beta_3\right)\|z\|^2\\ \overset{a.e.}{\leq}\ & -(\lambda-\frac{\zeta_1^2+\zeta_8^2}{4k_2}-\beta_3)\|z\|^2 \overset{a.e.}{\leq} -U(y), \end{aligned} \tag{34}$$

where $U(y)=c\|z\|^2$, for some positive constant c, is a continuous positive semi-definite function, and λ is defined in (29). The inequalities in (31) and (34) show that $V_L(y)\in\mathcal{L}_\infty$; hence, $\tilde{x}(t),\ e(t),\nu(t),\ \eta(t),\ r_{es}(t),\ r_{tr}(t),\ P(t)$ and $Q(t)\in\mathcal{L}_\infty$. Using (4) and (4), it can be shown that $\dot{\tilde{x}}(t)$, $\dot{e}(t)\in\mathcal{L}_\infty$. Based on the assumption that $x_d(t),\dot{x}_d(t)\in\mathcal{L}_\infty$, and $e(t),\ \dot{e}(t)\in\mathcal{L}_\infty$, $x(t),\ \dot{x}(t)\in\mathcal{L}_\infty$ by (3); moreover, using (3) and $\tilde{x}(t),\ \dot{\tilde{x}}(t)\in\mathcal{L}_\infty$, $\hat{x}(t),\ \dot{\hat{x}}(t)\in\mathcal{L}_\infty$. Based on Assumptions 2 and 5, the projection algorithm in (7), the boundedness of the $sgn(\cdot)$ and $\sigma(\cdot)$ functions, and $x_d(t)$, $\dot{x}_d(t),\ \hat{x}(t),\ \dot{\hat{x}}(t)\in\mathcal{L}_\infty$, the control input $u(t)$ is bounded from (20). Similarly, $\dot{\nu}(t),\dot{\eta}(t),\dot{r}_{es}(t),\dot{r}_{tr}(t)\in\mathcal{L}_\infty$ by using (5), (8), (9), (22); hence $\dot{z}(t)\in\mathcal{L}_\infty$, using (17); hence, $U(y)$ is uniformly continuous. It can be concluded that $c\|z\|^2\to 0$ as $t\to\infty$. Using the definition of $z(t)$ in (17), it can be shown that $\|\tilde{x}\|,\|e\|,\|r_{es}\|,\|r_{tr}\|,\|\nu\|,\|\eta\|\to 0$ as $t\to\infty$. Using (4), and standard linear analysis, it can be further shown that $\left\|\dot{\tilde{x}}\right\|\to 0$ as $t\to\infty$. ■

VI. Experiment Results

The performance of the proposed output feedback control method is tested on a two-link robot manipulator, where two aluminum links are mounted on a 240 Nm (first link) and a 20 Nm (second link) switched reluctance motor. The motor resolvers provide rotor position measurements with a resolution of 614400 pulses/revolution. Data acquisition and control implementation were performed in real-time using QNX at a frequency of 1.0 kHz. The two-link revolute robot is modeled with the following dynamics

$$M(x)\ddot{x}+V_m(x,\dot{x})\dot{x}+F(\dot{x})+\tau_d(t)=u(t), \tag{35}$$

where $x=\begin{bmatrix}x_1 & x_2\end{bmatrix}^T$ are the angular positions (rad), $\dot{x}=\begin{bmatrix}\dot{x}_1 & \dot{x}_2\end{bmatrix}^T$ are the angular velocities (rad/s) of the two links respectively, $M(x)\in\mathbb{R}^{2\times 2}$ is the inertia matrix, $V_m(x,\dot{x})\in\mathbb{R}^{2\times 2}$ denotes the centripetal-Coriolis matrix, $F(\dot{x})\in\mathbb{R}^2$ denotes friction, and $\tau_d(t)\in\mathbb{R}^2$ is the external disturbance. The system in (35) can be rewritten as $\ddot{x}=f(x,\dot{x})+G(x,\dot{x})u+d$, where $f(x,\dot{x})\in\mathbb{R}^2$ and $G(x)\in\mathbb{R}^{2\times 2}$ are defined as $f(x,\dot{x})=-M^{-1}(V_m\dot{x}+F)$, $G(x)=M^{-1}$. The desired trajectory for each link of the manipulator is given as (in degrees) $x_{1d}=30\sin(1.5t)(1-\exp(-0.01t^3))$, $x_{2d}=30\sin(2.0t)(1-\exp(-0.05t^3))$. The control gains are chosen as $k=diag(25,\ 90)$, $\alpha=diag(22,\ 30)$, $\beta_1=\beta_2=0.2$, and $\Gamma_w=0.2\mathbb{I}_{8\times 8}$, $\Gamma_{v1}=\Gamma_{v2}=0.2\mathbb{I}_{2\times 2}$, where $\mathbb{I}_{n\times n}$ denotes an identity matrix of appropriate dimensions. The NNs was implemented with 7 hidden layer neurons and the neural network weights are initialized as uniformly distributed random numbers in the interval $[0.1,0.3]$. The initial conditions of the system and the observer were selected as $x(t)=\dot{x}(t)=[0\ 0]^T$, and $\hat{x}(t)=\dot{\hat{x}}(t)=[0\ 0]^T$, respectively.

The performance of the proposed output feedback controller is compared with two controllers: a classical PID controller, and the discontinuous OFB controller in [13]. A standard backwards difference algorithm is used to numerically determine velocity from the encoder readings to implement the PID controller. Control gains for the discontinuous controller in [13] were selected as $K_1=0.2$, $K_2=diag(410,38)$, and control gains for the PID controller were selected as $K_d=diag(120,30)$, $K_p=diag(750,90)$, and $K_i=diag(650,100)$. Table I shows the RMS and peak errors and torques of Links 1 and 2 at steady-state for all methods. The developed controller is shown to exhibit lower tracking errors with less control authority than the comparative controllers. Moreover, the DNN-based observer yields better velocity estimation in comparison with the high frequency content results from a backwards difference method as depicted in Fig. 1. Hence, the experiments illustrate that using the velocity estimation from a DNN-based observer, which adaptively compensates for unknown uncertainties in the system, results in improved control performance with lower frequency content than the compared methods.

VII. Conclusion

A DNN observer-based output feedback control of a class of second-order nonlinear uncertain systems is developed. The DNN-based observer works in conjunction with a dynamic filter to estimate the unmeasurable state. The DNN is

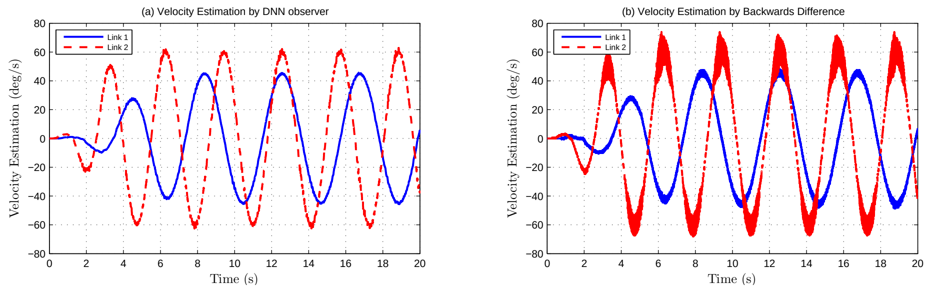

Fig. 1. Velocity estimation $\dot{x}(t)$ using (a) DNN-based observer and (b) numerical backwards difference.

TABLE I
STEADY-STATE RMS ERRORS AND TORQUES FOR EACH OF THE ANALYZED CONTROL DESIGNS.

	SSRMS e_1	SSRMS e_2	Max $\lvert e_1 \rvert$	Max $\lvert e_2 \rvert$	SSRMS τ_1	SSRMS τ_2	Max $\lvert \tau_1 \rvert$	Max $\lvert \tau_2 \rvert$
Classical PID	0.4538	0.2700	0.7371	0.5267	6.5805	2.4133	14.5871	9.0015
Robust OFB [13]	0.3552	0.2947	0.5819	0.6429	8.6509	1.2585	56.5796	4.6107
Proposed	0.1743	0.1740	0.3100	0.3760	6.3484	0.6944	12.5562	2.2122

updated on-line by weight update laws based on the estimation error, tracking error, and filter output. The controller is a combination of the NN feedforward term, and the estimated state feedback and sliding mode terms. Global asymptotic estimation of the unmeasurable state and global asymptotic tracking results are achieved, simultaneously. Results from an experiment using a two-link robot manipulator demonstrate the performance of the proposed output feedback controller.

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