Compensating for Fatigue-induced Time-varying Delayed Muscle Response in Neuromuscular Electrical Stimulation Control

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Abstract Neuromuscular electrical stimulation (NMES), often called functional electrical stimulation (FES), is a prescribed treatment for various neuromuscular disorders. When applied to articulate a person's limb, the respective skeletal muscle groups are known to rapidly fatigue compared to muscles activated by the nervous system. Recent results have shown that muscles have a delayed response to electrical stimulation, and more recent results indicate that this delayed response increases as the muscle fatigues. A NMES control method is developed in this chapter as a means to compensate for the varying input delay for the uncertain nonlinear dynamics for the lower limb. Experimental results are provided to demonstrate the performance of the developed controller.

1 Introduction

Neuromuscular electrical stimulation (NMES) is the use of electric current to activate skeletal muscle, typically applied by electrodes placed on the surface of the skin. NMES is commonly used in rehabilitative settings where the goal is to increase muscle size, strength, and function [1–4] and may also be used to produce functional tasks (e.g., standing, stepping, reaching, grasping, cycling) [5–9] where it is termed functional electrical stimulation (FES). Various feedback-based NMES controllers have been developed [10–22]; however, results that consider the muscle's delayed response to electrical stimulation, known as electromechanical delay (EMD), are not common. EMD may lead to degraded performance and instability, motivating the need for control designs that compensate for delay.

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Time-delays are prevalent in many engineering systems and have been well documented in literature (cf. [23–26] and recent monographs such as [27–32]). Few mathematical tools exist that can be used to develop controllers that compensate for input delays. Of these tools (namely Smith predictors [33], Artstein model reduction [34], and finite spectrum assignment [35]), few variations have been developed that can compensate for uncertain nonlinear systems. Methods which solve the input delay problem for uncertain nonlinear systems with known and unknown constant time-delays have been studied in [16, 36–43].

Based on the development in [41] for general Euler-Lagrange systems, the results in [16] developed a model-free robust controller that enabled the leg shank of a healthy normal volunteer to track a desired angular trajectory about the knee with a uniformly ultimately bounded error despite a known constant EMD. Motivated by [16], a state predictive hybrid control approach was developed in [43] that considered sampled state measurements and limb constraints in addition to the known constant EMD. Assuming exact knowledge of the limb dynamics, the result in [43] yields exponential tracking. Although results such as [16] and [43] provide insights on NMES in the presence of EMD, significant changes in muscle EMD were reported in [44–47] during voluntary fatiguing exercises. Further, NMES is well known to induce significant fatigue in contrast to volitional contractions. There are a number of suggested causes of NMES-induced fatigue [48,49] and efforts have been made to prevent or slow the onset of fatigue [50–64]. However, NMES-induced fatigue and the resulting time-varying EMD is of key importance when developing NMES controllers.

Control methods for time-varying input-delayed systems with linear plant models have been studied extensively. Discrete predictor-based techniques have been developed for linear systems with time-varying input delay in [65], where small bounded uncertainties in the system parameters, delay, and sampling instants are considered. A delayed feedback controller was developed in [66] for uncertain linear systems with a time-varying input delay based on a reduction method. A robust control method for uncertain linear systems with time-varying input delays was developed in [67], which combines a novel Lyapunov-Krasovskii (LK) functional and a neutral transformation to obtain sufficient conditions for closed-loop robustness. Predictive controllers have also been developed under the assumption that input delay systems can be represented by hyperbolic partial differential equations (cf. [25, 28] and references therein). This fact is exploited in [68] to design controllers for actuator delayed linear systems where the time delayed system is modeled as an ordinary differential equation (ODE) - partial differential equation (PDE) cascade using an infinite dimensional transformation where the non-delayed input acts at the PDE boundary. Linearized controllers have been developed for nonlinear systems (cf. [69, 70]), but because the stability of the closed loop system is only valid within a region around the point of linearization, a complete nonlinear control solution to the time-varying input delay problem is still motivated.

A finite-time stabilizing controller developed in [71] compensates for time-varying input delays in nonlinear systems with triangular structures using an integrator backstepping technique. More recently, Bekiaris-Liberis and Krstic [72] extended the results in [68] and [40] to develop a control method for forward complete nonlinear systems with time-varying input delays. Under the assumption of the existence of a stabilizing controller in the absence of the input delay, an invertible infinite dimensional backstepping transformation is used to yield an asymptotically stable system in the presence of a time-varying input delay. While these results have been successful for certain classes of nonlinear input-delayed systems, the applicability of the methods to general uncertain Euler-Lagrange dynamics is not clear. Motivated by this issue, [73] provided a transformation to convert an Euler-Lagrange system into a forward-complete system, but such a transformation requires exact model knowledge of the Euler-Lagrange dynamics; thus, the technique is not applicable when the system parameters are unknown or the dynamics are uncertain. This implies that methods developed for forward-complete systems with input delays may not be applicable to uncertain Euler-Lagrange systems.

In this chapter, a control method is developed to compensate for time-varying EMD during NMES where the muscle dynamics are uncertain, nonlinear, and contain additive disturbances, under the assumption that the known time-delay is bounded and slowly varying. As in our previous work, LK functionals are used to facilitate the design and analysis of a control method that can compensate for the input delay. Since the LK functionals contain time-varying delay terms, additional complexities are introduced into the analysis. Techniques used to compensate for the time-varying delay result in new sufficient control conditions that depend on the length of the delay as well as the rate of delay. The developed controller achieves semi-global uniformly ultimately bounded tracking despite the time-varying input delay, parametric uncertainties and additive bounded disturbances in the dynamics. Experiments are provided to examine the performance of the developed controller.

2 Knee joint dynamics

The knee-joint dynamics are modeled as [15]

$$M_I(\ddot{q}) + M_e(q) + M_g(q) + M_v(\dot{q}) + \bar{d} = \mu, \tag{1}$$

where $M_I : \mathbb{R} \to \mathbb{R}$ denotes the inertial effects of the shank-foot complex about the knee-joint, $M_e : \mathbb{R} \to \mathbb{R}$ denotes the elastic effects due to joint stiffness, $M_g : \mathbb{R} \to \mathbb{R}$ denotes the gravitational component, $M_v : \mathbb{R} \to \mathbb{R}$, denotes the viscous effects due to damping in the musculotendon complex, $\bar{d} \in \mathbb{R}$ is an unknown bounded time-varying disturbance from unmodeled dynamics, $\mu \in \mathbb{R}$ denotes the torque produced at the knee-joint due to stimulation, and $\tau \in \mathbb{R}$ denotes the EMD. The inertial and gravitational effects in (1) are modeled as

$$M_I(\ddot{q}) \triangleq J\ddot{q}, \quad M_g(q) \triangleq mgl\sin(q), \tag{2}$$

where $J, m, g, l \in \mathbb{R}$ are positive constants and $q, \dot{q}, \ddot{q} \in \mathbb{R}$ denote the angular position, velocity, and acceleration of the shank about the knee-joint, respectively. The terms J, m, and l denote the unknown inertia of the combined shank and foot, the unknown combined mass of the shank and foot, and the unknown distance between the knee-joint and the lumped center of mass of the shank and foot, respectively, while g denotes the gravitational acceleration. The elastic and viscous effects are modeled as

$$M_e(q) \triangleq k_1(\exp(-k_2 q))(q - k_3), \tag{3}$$

where $k_1, k_2, k_3 \in \mathbb{R}$ are unknown positive constants and

$$M_v(\dot{q}) \triangleq -B_1 \tanh(-B_2 \dot{q}) + B_3 \dot{q}, \tag{4}$$

where $B_1, B_2, B_3 \in \mathbb{R}$ are unknown positive constants. The subsequent development is based on the assumption that q and $\dot{q}$ are measurable outputs. Throughout the paper, a time-dependent delayed function is denoted as

$$(\cdot)_\tau(t) \triangleq \begin{cases} (\cdot)(t - \tau(t)) & t - \tau(t) \geq t_0 \\ 0 & t - \tau(t) < t_0 \end{cases}, \tag{5}$$

where $t_0 \in \mathbb{R}$ is the initial time. Additionally, let $\|\cdot\|$ denote the Euclidean norm of a vector.

The torque produced about the knee is controlled through muscle forces that are elicited by NMES/FES. For simplicity (and without loss of generality), the subsequent development focuses on producing knee torque through muscle tendon forces generated by electrical stimulation of the quadriceps. The total muscle force is a net sum of active force generated by contractile, elastic, and viscous elements [15]. The muscle force generated at the tendon is the projection of net sum of these elements along the line parallel to the tendon. The force development in the muscle is delayed due to the finite propagation time of chemical ions such as Ca^{2+} and action potential along the T-tubule system, cross-bridge formation between actin and myosin filaments, the subsequent tension development, and the stretching of the series elastic components by the contractile components in the muscle [46, 74, 75]. The EMD is influenced by factors such as fatigue, rate of force production, and types of muscle contractions. The total muscle force generated at the tendon, denoted by $F \in \mathbb{R}$, is defined as

$$F \triangleq \xi(q, \dot{q}) u_\tau. \tag{6}$$

In (6), $\xi : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ denotes an unknown nonlinear function of the muscle length and velocity, and the applied voltage potential across the quadriceps muscle, denoted by $u_\tau \in \mathbb{R}$, includes the time-delay to capture the latency that is present between the application of voltage and force production [13, 76]. The introduction of the unknown nonlinear function ξ enables the muscle contraction to be considered under general dynamic conditions in the subsequent control development. The uncertain and unknown function ξ captures the dynamic characteristics of muscle recruitment (approximated by a continuously differentiable function), muscle force-length and muscle force-velocity relationships, and active and passive muscle characteristics [15]. The knee torque is related to the muscle tendon force as

$$\mu = \zeta(q)F, \tag{7}$$

where $\zeta : \mathbb{R} \to \mathbb{R}$ denotes a positive moment arm that changes with the extension and flexion of the leg [77, 78].

The model developed in (1)-(7) is used to examine the stability of the subsequently developed controller, but the controller does not explicitly depend on these models. The following assumptions and notations are used to facilitate the subsequent control development and stability analysis.

Assumption 1. The moment arm ζ is assumed to be a non-zero, positive, bounded function [77, 78] whose first two time derivatives exist and are bounded. Based on the empirical data [79, 80], the function ξ is assumed to be a non-zero, positive, and bounded function with bounded first and second time derivatives.

For notational brevity, an auxiliary non-zero unknown scalar function $\Omega : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is defined as

$$\Omega(q, \dot{q}) \triangleq \zeta(q)\, \xi(q, \dot{q}). \tag{8}$$

From Assumption 1, the first and second time derivatives of Ω exist and are bounded.

Assumption 2. The unknown disturbance $\bar{d}$ is bounded and its first and second derivatives with respect to time exist and are bounded**.** Based on Assumption 1, the ratio $\bar{d}/\Omega(q, \dot{q})$ denoted by d is also bounded and its first and second derivatives with respect to time exist and are bounded.

Assumption 3. The time delay and its first and second time derivatives are bounded such that $0 \leq \tau(t) \leq \varphi_1$, $|\dot{\tau}(t)| < \varphi_2 < 1$, and $|\ddot{\tau}(t)| \leq \varphi_3$, for all $t \in \mathbb{R}_{\geq 0}$, where $\varphi_1, \varphi_2, \varphi_3 \in \mathbb{R}$ are known positive constants.

The implications of Assumptions 2 and 3 are that the disturbance and delay are sufficiently smooth and that the delay is sufficiently slow. The development of an input-delayed controller for arbitrarily fast time-varying delays remains an open problem for NMES with unknown dynamics.

Combining (1)-(8), the knee joint dynamics can be expressed as

$$M(q, \dot{q})\ddot{q} + f(q, \dot{q}) + d = u_\tau, \tag{9}$$

where $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \triangleq \frac{M_e + M_g + M_v}{\Omega}$, $M : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \triangleq \frac{J}{\Omega}$, and $d \triangleq \frac{\bar{d}}{\Omega}$. Based on Assumption 1, M can be bounded as

$$\underline{m} \leq |M(x_1, x_2)| \leq \overline{m} \tag{10}$$

for all $x_1, x_2 \in \mathbb{R}$, where $\underline{m}, \overline{m} \in \mathbb{R}$ are known positive constants.

3 Control objective

The objective is to design a continuous controller that will ensure the generalized state q of the input-delayed system in (9) tracks a desired trajectory despite uncertainties and additive bounded disturbances in the dynamic model. To quantify the control objective, a tracking error denoted by $e \in \mathbb{R}$, is defined as

$$e \triangleq q_d - q, \tag{11}$$

where $q_d \in \mathbb{R}$ denotes the desired trajectory and is designed such that $q_d, \dot{q}_d, \ddot{q}_d \in \mathscr{L}_\infty$. To facilitate the subsequent analysis, a measurable auxiliary tracking error, denoted by $r \in \mathbb{R}$, is defined as

$$r \triangleq \dot{e} + \alpha e - Be_z, \tag{12}$$

where $\alpha \in \mathbb{R}^+$ is a known constant control gain, and $B \in \mathbb{R}^+$ is a known constant best guess estimate of M^{-1}. In (12), $e_z \in \mathbb{R}$ is an auxiliary signal containing the time-delay in the system, defined as

$$e_z \triangleq \int_{t-\tau(t)}^{t} u(\theta)\,\mathrm{d}\theta. \tag{13}$$

The error between B and M^{-1} is denoted by $\eta : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ and is defined as

$$\eta(q,\dot{q}) \triangleq B - \frac{1}{M(q,\dot{q})} \tag{14}$$

and satisfies

$$|\eta(x_1,x_2)| \leq \overline{\eta} \tag{15}$$

for all $x_1, x_2 \in \mathbb{R}$, where $\overline{\eta} \in \mathbb{R}^+$ is a known constant.

4 Control development

The open-loop error system can be obtained by multiplying the time derivative of (12) by M and utilizing the expressions in (9), (11), (13) and (14) to yield

$$\begin{aligned} M(q,\dot{q})\dot{r} = {} & M(q,\dot{q})\ddot{q}_d + f(q,\dot{q}) + d + \alpha M(q,\dot{q})\dot{e}. \\ & - M(q,\dot{q})\eta(q,\dot{q})(u - u_\tau + u_\tau\dot{\tau}) - u - u_\tau\dot{\tau}. \end{aligned} \tag{16}$$

For notational brevity, the dependance of M and η on q and $\dot{q}$ is omitted henceforth. Based on the error system formulation in (12) and (13), the open-loop error system in (16) contains a delay-free control input. From (16) and the subsequent stability analysis, the control input is designed as

$$u = k_b r, \tag{17}$$

where $k_b \in \mathbb{R}$ is a known positive constant control gain. To facilitate the subsequent stability analysis, an auxiliary signal $N_d \in \mathbb{R}^n$ is defined as

$$N_d \triangleq M_d \ddot{q}_d + f_d, \tag{18}$$

where $M_d \triangleq M(q_d, \dot{q}_d)$ and $f_d \triangleq f(q_d, \dot{q}_d)$.

The closed-loop error system is obtained by adding and subtracting N_d and e to (16) and utilizing (12) and (17) to yield

$$\begin{aligned} M\dot{r} = &-\frac{1}{2}\dot{M}r + \chi + S - k_b r - k_b M \eta \left(r - r_\tau + r_\tau \dot{\tau}\right) \\ &- k_b r_\tau \dot{\tau} - e, \end{aligned} \tag{19}$$

where the auxiliary terms χ, $S \in \mathbb{R}^n$ are defined as

$$\begin{aligned} \chi \triangleq &\frac{1}{2}\dot{M}r + e + (M - M_d)\ddot{q}_d + f(q, \dot{q}) - f_d \\ &+ \alpha M (r - \alpha e + B e_z). \end{aligned} \tag{20}$$

$$S \triangleq N_d + d. \tag{21}$$

Using Assumption 2, the following inequality can be developed based on the expression in (21)

$$\|S\| \leq \bar{s}, \tag{22}$$

where $\bar{s} \in \mathbb{R}^+$ is a known constant. The structure of (19) is motivated by the desire to segregate terms that can be upper bounded by state-dependent terms and terms that can be upper bounded by constants. Using the Mean Value Theorem, the expression in (20) can be upper bounded as

$$\|\chi\| \leq \rho(\|z\|)\|z\|, \tag{23}$$

where $\rho(\|z\|)$ is a positive and strictly increasing function and $z \in \mathbb{R}^3$ is defined as

$$z \triangleq \left[\, e \;\, r \;\, e_z \,\right]^T. \tag{24}$$

5 Stability analysis

To facilitate the subsequent stability analysis, let $y \in \mathbb{R}^4$ be defined as

$$y \triangleq \left[\, e \;\, r \;\, \sqrt{P} \;\, \sqrt{Q} \,\right]^T, \tag{25}$$

where the signals $P, Q \in \mathbb{R}$ are defined as

$$P \triangleq \omega \int_{t-\tau(t)}^{t} \left(\int_{s}^{t} u^2(\theta)\, \mathrm{d}\theta \right) \mathrm{d}s, \tag{26}$$

$$Q \triangleq \frac{k_b\left(2\overline{m}\overline{\eta} + \varphi_2\right)}{2\left(1-\dot{\tau}\right)} \int_{t-\tau(t)}^{t} \left\| r(\theta) \right\|^2 \mathrm{d}\theta, \tag{27}$$

where $\omega \in \mathbb{R}$ is a known positive constant. Let the auxiliary constants σ, δ, and γ be defined as

$$\sigma = \min\left\{ \frac{\alpha}{4}, \frac{k_b}{8}, \frac{\omega\left(1-\varphi_2\right)}{8\varphi_1} \right\}, \tag{28}$$

$$\delta = \min\left\{ \frac{\alpha}{4}, \frac{k_b}{8}, \frac{\left(1-\varphi_2\right)}{4\varphi_1}, \frac{\omega k_b\left(1-\varphi_2\right)^2}{\left(4\overline{m}\overline{\eta}+2\varphi_2\right)} \right\}, \tag{29}$$

$$\gamma = \max\left\{ 1, \sqrt{\frac{4k_b\varphi_1}{2\overline{m}\overline{\eta}+\varphi_2}} \right\}, \tag{30}$$

and let

$$\mathscr{D} \triangleq \left\{ x \in \mathbb{R}^4 \mid \|x\| \leq \frac{1}{\gamma} \inf\left\{ \rho^{-1}\left[\sqrt{k_b\sigma}, \infty\right) \right\} \right\},$$

$$\mathscr{S}_{\mathscr{D}} \triangleq \left\{ x \in \mathscr{D} \mid \|x\| < \sqrt{\frac{\phi_1}{\gamma^2\phi_2}} \inf\left\{ \rho^{-1}\left[\sqrt{k_b\sigma}, \infty\right) \right\} \right\}.$$

Theorem 1. *Given the dynamics in* (9)*, provided the control gains are selected based on the sufficient conditions*

$$\alpha > B, \;\; \omega > \frac{4B\varphi_1}{\left(1-\varphi_2\right)}, \tag{31}$$

and the input delay τ, its time derivatives $\dot{\tau}$ and $\ddot{\tau}$, and the inertia estimate mismatch η are small enough so that there exists a positive gain $k_b \in \mathbb{R}$ that satisfies

$$k_b > \frac{2\varphi_3\left(2\overline{m}\overline{\eta}+\varphi_2\right)}{\omega\left(1-\varphi_2\right)^3}, \tag{32}$$

$$\varphi_1 < \frac{1}{\omega k_b}\left(\frac{1}{4} - \frac{\left(2\overline{\eta}\overline{m}+\varphi_2\right)}{1-\varphi_2} \right), \tag{33}$$

$$\frac{\phi_2\bar{s}^2\gamma^2}{\phi_1\delta k_b} < \left(\inf\left\{ \rho^{-1}\left[\sqrt{k_b\sigma}, \infty\right) \right\} \right)^2, \tag{34}$$

the controller in (17) *ensures uniformly ultimately bounded tracking in the sense that* $\limsup_{t\to\infty}\|y(t)\| \leq \sqrt{\frac{\phi_2\bar{s}^2}{\phi_1\delta k_b}}$*, for all $y(t_0) \in \mathscr{S}_{\mathscr{D}}$, and the convergence to the ultimate bound is exponential.*

Proof. Let $V_L : \mathscr{D} \times [0, \infty) \to \mathbb{R}$ be a continuously differentiable, positive-definite function defined as

$$V_L \triangleq \frac{1}{2}e^T e + \frac{1}{2}r^T M r + P + Q, \tag{35}$$

which can be bounded as

$$\phi_1 \|y\|^2 \le V_L \le \phi_2 \|y\|^2 \tag{36}$$

where the constants $\phi_1, \phi_2 \in \mathbb{R}$ are defined as

$$\phi_1 \triangleq \frac{1}{2} min\left[\underline{m}, 1\right], \quad \phi_2 \triangleq max\left[\frac{1}{2}\overline{m}, 1\right]. \tag{37}$$

After utilizing (12) and (19), applying the Leibniz Rule to determine the time derivative of (26) and (27), and by canceling similar terms, the time derivative of (35) can be expressed as

$$\begin{aligned}\dot{V}_L = & -\alpha e^2 - k_b r^2 + Bee_z + r\chi + rS - k_b \eta M r^2 \\ & + k_b (1-\dot{\tau}) \eta M r r_\tau - k_b r r_\tau \dot{\tau} + \omega \tau k_b^2 r^2 \\ & - \omega (1-\dot{\tau}) \int_{t-\tau(t)}^{t} u^2(\theta)\, d\theta \\ & + \frac{\ddot{\tau}\left(2\overline{m}\overline{\eta} + \varphi_2\right)}{2k_b (1-\dot{\tau})^2} \int_{t-\tau(t)}^{t} u^2(\theta)\, \mathrm{d}\theta \\ & + \frac{k_b\left(2\overline{m}\overline{\eta} + \varphi_2\right)}{2(1-\dot{\tau})} r^2 - \frac{k_b\left(2\overline{m}\overline{\eta} + \varphi_2\right)}{2} r_\tau^2. \end{aligned} \tag{38}$$

By utilizing Young's inequality, Assumption 3, (15), (17), (22), and (23), (38) can be expanded, regrouped and upper bounded as

$$\begin{aligned}\dot{V}_L \le & -\alpha e^2 - k_b r^2 + \rho(\|z\|)\|z\||r| + |r|\overline{s} \\ & + k_b \frac{\left(2\overline{\eta}\overline{m} + \varphi_2\right)}{1-\varphi_2} r^2 + \omega \varphi_1 k_b^2 r^2 \\ & + \frac{B}{2} e^2 + \frac{B}{2} e_z^2 - \omega (1-\varphi_2) \int_{t-\tau(t)}^{t} u^2(\theta)\, d\theta \\ & + \frac{\varphi_3\left(2\overline{m}\overline{\eta} + \varphi_2\right)}{2k_b (1-\varphi_2)^2} \int_{t-\tau(t)}^{t} u^2(\theta)\, \mathrm{d}\theta. \end{aligned} \tag{39}$$

Utilizing the Cauchy-Schwartz inequality and (13) yields

$$\|e_z\|^2 \le \tau \int_{t-\tau(t)}^{t} \|u(\theta)\|^2 \mathrm{d}\theta. \tag{40}$$

Using (40) and the inequality [41]

$$\int_{t-\tau(t)}^{t} \left(\int_{s}^{t} \|u(\theta)\|^2 \mathrm{d}\theta\right) \mathrm{d}s \le \tau \int_{t-\tau(t)}^{t} \|u(\theta)\|^2 \mathrm{d}\theta$$

the expression in (39) can be bounded as

$$\begin{aligned}\dot{V}_L \leq & -\frac{\alpha}{4}e^2 - \frac{k_b}{8}r^2 - \frac{\omega(1-\varphi_2)}{8\varphi_1}e_z^2 + \frac{\rho^2(\|z\|)}{k_b}\|z\|^2 \\ & -\frac{\alpha}{4}e^2 - \frac{k_b}{8}r^2 - \frac{(1-\varphi_2)}{4\varphi_1}P - \frac{\omega k_b(1-\varphi_2)^2}{(4\overline{m}\overline{\eta}+2\varphi_2)}Q + \frac{\overline{s}^2}{k_b} \\ & -\left(\frac{\omega(1-\varphi_2)}{4} - \frac{\varphi_3(2\overline{m}\overline{\eta}+\varphi_2)}{2k_b(1-\varphi_2)^2}\right)\int_{t-\tau(t)}^{t} u^2(\theta)\,\mathrm{d}\theta \\ & -\left(\frac{\alpha}{2} - \frac{B}{2}\right)e^2 - \left(\frac{\omega(1-\varphi_2)}{8\varphi_1} - \frac{B}{2}\right)e_z^2 \\ & -k_b\left(\frac{1}{4} - \frac{(2\overline{\eta}\overline{m}+\varphi_2)}{1-\varphi_2} - \omega\varphi_1 k_b\right)r^2. \end{aligned} \tag{41}$$

Provided the sufficient gain conditions in (31)-(33) are satisfied, the inequality $\|z\| \leq \gamma\|y\|$ can be used to bound the expression in (41) as

$$\begin{aligned}\dot{V}_L &\leq -\left(\sigma - \frac{\rho^2(\gamma\|y\|)}{k_b}\right)\|z\|^2 - 2\delta\|y\|^2 + \frac{\overline{s}^2}{k_b}, \\ &\leq -\delta\|y\|^2,\ \forall y \in \mathscr{D},\ \|y\| > \frac{\overline{s}}{\sqrt{\delta k_b}}, \end{aligned} \tag{42}$$

where σ and δ were introduced in (28) and (29), respectively. Using (34)-(37) and (42), Theorem 4.18 in [81] can be invoked to conclude that

$$\begin{aligned} & y(t) \in \mathscr{D}, \forall t \in [t_0, \infty)\ \forall y(t_0) \in \mathscr{S}_{\mathscr{D}}, \\ & \limsup_{t\to\infty}\|y(t)\| \leq \sqrt{\frac{\phi_2\overline{s}^2}{2\phi_1\delta k_b}},\ \forall y(t_0) \in \mathscr{S}_{\mathscr{D}}. \end{aligned} \tag{43}$$

Thus, $e, r, P, Q \in \mathscr{L}_\infty$, hence, using $\|z\| \leq \gamma\|y\|$, $\|z\| \in \mathscr{L}_\infty$, and hence, $e_u \in \mathscr{L}_\infty$. The closed-loop error system can be used to conclude that the remaining signals are bounded.

Using (37), for all $y(t_0) \in \mathscr{S}_{\mathscr{D}}$, the Lyapunov derivative in (42) can be bounded as

$$\dot{V}_L \leq -\frac{2\delta}{\phi_2}V_L + \frac{\overline{s}^2}{k_b}. \tag{44}$$

Solving the differential inequality, (44), the Lyapunov function can be bounded as

$$V_L(y(t), t) \leq \left(V_L(y(t_0), t_0) - \frac{\phi_2\overline{s}^2}{2\delta k_b}\right)e^{-\frac{2\delta(t_0-t)}{\phi_2}} + \frac{\phi_2\overline{s}^2}{2\delta k_b}. \tag{45}$$

Substituting for the Lyapunov function in (45) using the bound (37) yields

$$\|y(t)\|^2 \leq \left(\frac{\phi_2 \|y(t_0)\|^2}{\phi_1} - \frac{\phi_2 \bar{s}^2}{2\phi_1 \delta k_b} \right) e^{-\frac{2\delta(t_0 - t)}{\phi_2}} + \frac{\phi_2 \bar{s}^2}{2\phi_1 \delta k_b},$$

establishing exponential convergence of the tracking error to the ultimate bound.

6 Experiments

One able-bodied male (age 26) participated in the study to examine the performance of the developed delay compensation controller. The electrical stimulation responses of healthy subjects have been reported to be similar to those of paraplegic subjects [13, 17, 82, 83]. Therefore, a healthy subject was used as a substitute for affected individuals. Prior to participation, written informed consent was obtained from the individual, as approved by the institutional review board at the University of Florida.

All testing was performed using an apparatus that consists of a custom computer-controlled stimulation circuit and a leg extension machine (LEM; Fig. 1). The LEM includes optical encoders to measure the angle between the femur and the tibia. The LEM allows seating adjustments to ensure that the rotation of the knee is about the encoder axis and a mechanical stop was used to prevent hyperextension. A computer was used to collect data from the encoders and execute the closed-loop control algorithms. Voltage was applied with a pair of 3” by 5” oval PALS® Platinum surface electrodes placed over the distal–medial and proximal–lateral portions of the quadriceps femoris muscle group. Surface electrodes for the study were provided compliments of Axelgaard Manufacturing Co.

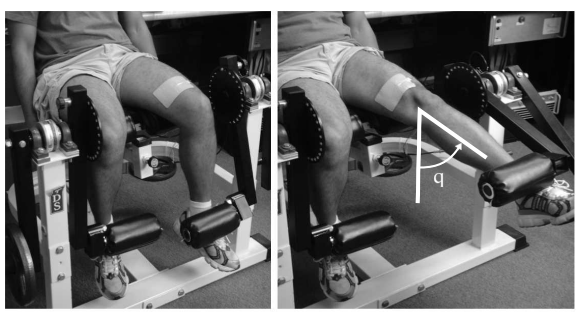

Fig. 1 The experimental setup includes a leg extension exercise machine, encoders to determine the subject’s leg angle, q(t), and a computer to control stimulation and gather data.

To better understand the effect of the delay compensation term Be_z in (12) the developed proportional-derivative delay compensation (PDDC) controller was compared to a controller of similar form that did not include the delay compensation term. In other words, the PDDC controller was compared to a proportional-derivative (PD) controller. During testing, the individual was instructed to relax as much as possible and to allow the stimulation to control the limb motion (i.e., the subject was not supposed to influence the leg motion voluntarily and was not allowed to see the desired trajectory). The desired trajectory was selected as a sinusoid ranging from 10° to 45° with a period of 2.5 seconds.

The desired trajectory also included a smooth step function (0° to 30° lasting 1 second) at equally spaced intervals to measure the EMD during the course of the experiment. The EMD was calculated as the difference between the time at the onset of stimulation and the time at which the leg angle increased by 0.005 radians. Measurement of the EMD during the PD controller trials allowed for modeling of the delay during the PDDC controller trials. As such, the PD trials were always performed prior to the PDDC trials. The EMD measurements were curve-fit as a function of time with a model of the form $\tau(t) = a\exp(bt) + c\exp(dt)$ and this model was used to calculate the control term e_z in (12).

A total of eight trials were conducted where the PD and PDDC controllers were examined twice for each leg. The PD controller was first examined where the control gains were tuned in short pretrial tests to reduce the tracking error. After gain tuning, two PD trials were completed where 5 minutes of rest was allowed between the trials and the same control gains were used in each of the trials. The individual was then allowed to rest for one hour before two PDDC trials were completed, where the same gain tuning and resting procedures were followed as in the PD trials.

One-way, paired t-tests were used to determine statistical differences between the two controllers in terms of the measured RMS and peak errors. The level of significance was set at $\alpha = 0.05$ for the t-tests.

7 Results

The RMS error and peak error were calculated during steady state and are listed in Tables 1 and 2, respectively. Steady state was defined by removing the first period of the desired sinusoid to be tracked (i.e., the transient period). T-tests indicate that the mean RMS and peak errors are statistically less for the PDDC controller with P-values of 0.001 and 0.009, respectively. An example trial run which compares the PD and PDDC controllers is shown in Figure 2 with a more detailed view in Figure 3. Example measurements and curve fit of the time-varying EMD are shown in Figure 4.

8 Discussion

The experimental results indicate that the developed PDDC controller results in statistically improved tracking performance when compared to the PD controller. Overall, the PDDC controller resulted a 47% reduction in RMS error and a 46% reduction in peak error (see Tables 1 and 2). While delay compensation was able to

Table 1 Steady state RMS error (degrees) of the two examined controllers. PD indicates a proportional-derivative controller while PDDC indicates the developed proportional-derivative controller with delay compensation.

Leg - Trial	PD	PDDC
Left - 1st	7.22	4.61
Left - 2nd	7.84	4.10
Right - 1st	7.40	3.01
Right - 2nd	6.91	3.70
MEAN	7.34	3.86*
SD	0.39	0.68

* Indicates statistically significant difference in the means (P-value = 0.001)

Table 2 Peak steady state errors (degrees) of the two examined controllers. PD indicates a proportional-derivative controller while PDDC indicates the developed proportional-derivative controller with delay compensation.

Leg - Trial	PD	PDDC
Left - 1st	18.78	10.17
Left - 2nd	24.30	9.68
Right - 1st	18.93	14.22
Right - 2nd	19.32	9.40
MEAN	20.33	10.86*
SD	2.65	2.26

* Indicates statistically significant difference in the means (P-value = 0.009)

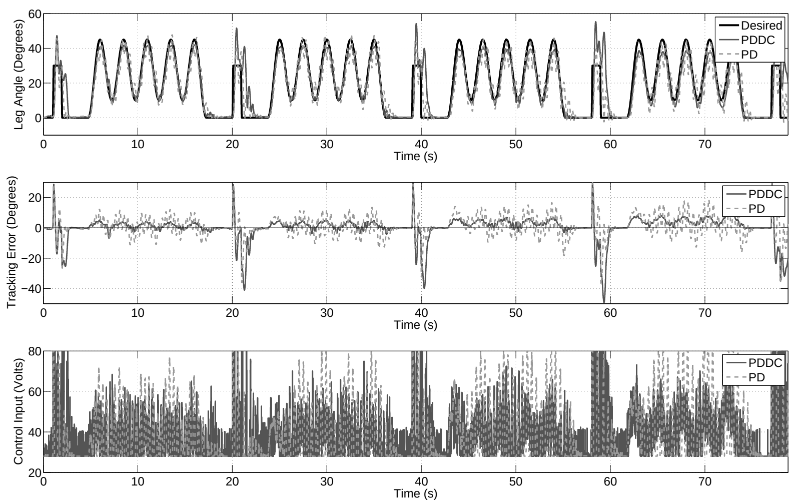

Fig. 2 Example tracking performance of the PD and PDDC controllers.

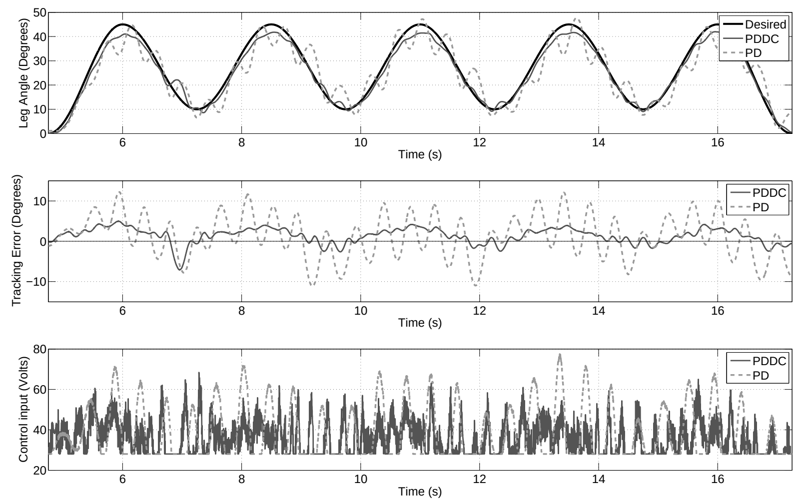

Fig. 3 Cropped example of the tracking performance of the PD and PDDC controllers.

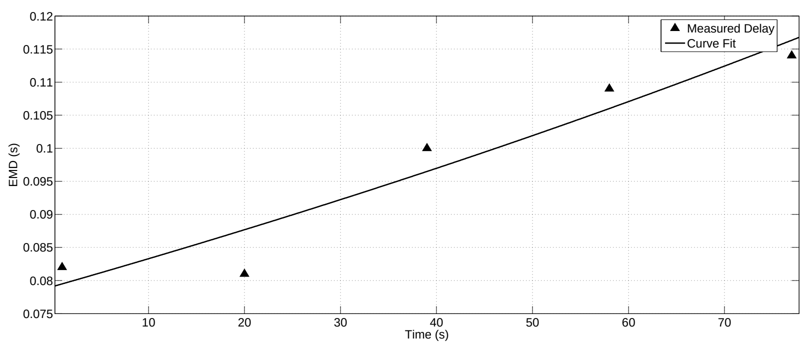

Fig. 4 Example of the time-varying EMD and curve fit.

improve the tracking performance, there are some limitations to the controller. The primary limitation to the PDDC controller is that it requires the time-varying EMD to be known. In the present experiments, an estimate of the time-varying EMD was computed by periodically placing step functions throughout the desired trajectory. In practice, if the controller is implemented during a functional activity (e.g., walking or cycling), an estimate of the delay would need to be calculated without interrupting the desired trajectory. One potential solution may be to use electromyography to estimate muscle fatigue as stimulus pulses are being delivered [84] and subsequently estimate the EMD as a function of fatigue. Another potential solution is to develop a controller which does not require knowledge of the time-varying EMD. RISE-based and neural network-based NMES controllers have demonstrated better performance without considering the EMD [15, 21] for short duration experiments; however, it is presently unclear how these controllers can be modified to compensate for EMD. In conclusion, future efforts should examine methods to estimate the time-varying EMD, methods to compensate for known time-varying EMD in alternative control structures, and methods to compensate for unknown time-varying EMD.

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