Saturated RISE Feedback Control for a Class of Second-Order Nonlinear Systems

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***Abstract*—A saturated controller is developed for a class of uncertain, second-order, nonlinear systems which includes time-varying and nonlinearly parameterized functions with bounded disturbances using a continuous control law with smooth saturation functions. Based on the robust integral of the sign of the error (RISE) control methodology, the developed controller is able to utilize the benefits of high gain control strategies while guaranteeing saturation limits are not surpassed. The bounds on the control are known a priori and can be adjusted by changing the feedback gains. The saturated controller yields asymptotic tracking despite model uncertainty and added disturbances in the dynamics. Experimental results using a two-link robot manipulator demonstrate the performance of the developed controller.**

I. Introduction

Robust, high gain controllers can be effective methods to compensate for nonlinear systems with unstructured parametric uncertainties and bounded disturbances. In general, robust control techniques (including all previous RISE methods) do not take into account the fact that the commanded input may require more actuation than is physically possible by the system (e.g., due to large initial condition offsets, an aggressive desired trajectory, or large perturbations). For example, the typical RISE structure uses a sufficiently large gain multiplied by an integral term, which could potentially exceed actuator capabilities under some conditions. Motivated by these issues, some efforts have focused on developing saturated controllers for the regulation problem (cf. [1]–[3]) and the more general tracking problem (cf. [4]–[10]). In [11], the authors developed an adaptive, full-state feedback controller to yield asymptotic tracking while compensating for unknown parametric uncertainties using multiple embedded hyperbolic saturation functions. The authors of [4] were able to extend the PID-based work of [1] to the tracking control problem by utilizing a general class of saturation functions to achieve a global uniform asymptotic tracking result for a linearly parameterizable system. The work was based on prior results in [12] and [5] which incorporated hyperbolic saturation functions into the saturated PD+ control strategy developed in [6]. The works of [4], [5], [12] exploit saturation-avoidance strategies. Anti-windup schemes have been developed in results such as [13] to compensate for saturation nonlinearities in nonlinear Euler-Lagrange systems using PID-like control structures. Results in [14] and [15] achieve global regulation of saturated nonlinear systems using a PID-like control structure and a passivity-based analysis. To compensate for uncertain dynamics and the evaluation of an unknown gravity term, the results in [7] includes an additional saturated integral term and uses energy shaping and damping injection methods to yield asymptotic tracking. More recently, a saturated PID controller was developed in [8] which uses sigmoidal functions to achieve global asymptotic regulation to a setpoint; however, it is unclear how the result can be extended to the tracking problem due to the inclusion of stationary values of an auxiliary signal. While each of the mentioned contributions developed saturated controllers with asymptotic tracking regulation, previous results have not been developed that include both uncertain dynamics *and* additive unmodeled disturbances. Control methods which include these considerations were developed by authors in [16] and [17] via saturated adaptive robust control (SARC) algorithms, which yield ultimately bounded tracking results.

Motivated to achieve an asymptotic result, Corradini, et. al developed a discontinuous saturated sliding mode controller [10] for linear plant models in the presence of bounded matched uncertainties. In [9], two control algorithms are developed for robust stabilization of spacecraft in the presence of control input saturation, parametric uncertainty, and external disturbances using a discontinuous variable structure control design. However, while each of these saturated robust techniques are able to address uncertain nonlinear systems with additive disturbances, the discontinuous nature of the results motivates the design of continuous saturated robust control techniques. Robust control designs utilizing nested saturation functions for uncertain feedforward nonlinear systems [18]–[20] can yield global asymptotic stability despite unmodeled dynamic disturbances.

Based on the preliminary work in [21] focused on Euler-Lagrange systems, this paper focuses on a new RISE-based closed-loop error system development that consists of a saturated, continuous tracking controller for a class of uncertain, second-order nonlinear systems which includes time-varying and nonlinearly parameterized functions and unmodeled dynamic effects. The technical challenge presented by this objective is the need to introduce saturation bounds on the integral signum term while maintaining its functionality to implicitly learn the system disturbances. To

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achieve the result, a new auxiliary filter structure is designed using hyperbolic functions that work in tandem with the redesigned continuous saturated RISE-like control structure. While the controller is continuous, the closed-loop error system contains discontinuities which are examined through a differential inclusion framework. The resulting controller is bounded by the magnitude of an adjustable control gain, and yields asymptotic tracking.

II. Dynamic System

Consider a class of second order MIMO nonlinear systems of the following form[1]:

$$\ddot{x} = f(x, \dot{x}, t) + u(x, \dot{x}, t) + d(t) \tag{1}$$

where $x, \dot{x} \in \mathbb{R}^n$ are the generalized system states, $u \in \mathbb{R}^n$ is the generalized control input, $f : \mathbb{R}^n \times \mathbb{R}^n \times [0, \infty) \to \mathbb{R}^n$ is an unknown nonlinear C^2 function, and $d \in \mathbb{R}^n$ denotes a generalized, sufficiently smooth, nonvanishing nonlinear disturbance (e.g., unmodeled effects).

The subsequent development is based on the assumption that x and $\dot{x}$ are measurable outputs. Additionally, the following assumptions will be exploited.

Assumption 1. The nonlinear disturbance term and its first two time derivatives (i.e., $d, \dot{d}, \ddot{d}$) exist and are bounded by known constants.[2]

Assumption 2. The desired trajectory $x_d \in \mathbb{R}^n$ is designed such that $x_d^{(i)} \in \mathbb{R}^n$, $\forall i = 0, 1, ..., 4$ exist and are bounded.[3]

Remark 1. To aid the subsequent control design and analysis, the vector $Tanh(\cdot) \in \mathbb{R}^n$ and the matrix $Cosh(\cdot) \in \mathbb{R}^{n \times n}$ are defined as

$$Tanh(\xi) \triangleq [tanh(\xi_1), ..., tanh(\xi_n)]^T \tag{2}$$

$$Cosh(\xi) \triangleq diag\{cosh(\xi_1), ..., cosh(\xi_n)\} \tag{3}$$

where $\xi = [\xi_1, ..., \xi_n]^T \in \mathbb{R}^n$. Based on these definitions, the following inequalities hold $\forall \xi \in \mathbb{R}^n$ [25]:

$$\|\xi\|^2 \geq \sum_{i=1}^{n} ln(cosh(\xi_i)) \geq \frac{1}{2} tanh^2(\|\xi\|),$$

$$\|\xi\| > \|Tanh(\xi)\|, \quad \|Tanh(\xi)\|^2 \geq tanh^2(\|\xi\|),$$

$$\xi^T Tanh(\xi) \geq Tanh^T(\xi) Tanh(\xi). \tag{4}$$

Throughout the paper, $\|\cdot\|$ denotes the standard Euclidean norm.

[1]The result in this paper can be extended to n^{th}-order nonlinear systems following a similar development to those presented in [22], [23].

[2]Many practical disturbance terms are continuous including friction (see [24]), wind disturbances, wave/ocean disturbances, unmodeled sufficiently smooth disturbances, etc.

[3]Many guidance and navigation applications utilize smooth, high-order differentiable desired trajectories. Curve fitting methods can also be used to generate sufficiently smooth time-varying trajectories.

III. Control Objective

The objective is to design an amplitude-limited, continuous controller which ensures the system state x tracks a desired trajectory x_d. To quantify the control objective, a tracking error denoted by $e_1 \in \mathbb{R}^n$ is defined as

$$e_1 \triangleq x_d - x. \tag{5}$$

Embedding the control in a bounded trigonometric term is an obvious way to limit the control authority below an a priori limit; however, by injecting these terms, difficulty arises in the closed-loop stability analysis. This challenge is exacerbated by the presence of integral control functions that are included to compensate for added disturbances as in this result. Motivated by these stability analysis complexities and through an iterative analysis procedure, two measurable filtered tracking errors are designed which include smooth saturation terms. Specifically, the filtered tracking errors $e_2, r \in \mathbb{R}^n$, are defined as

$$e_2 \triangleq \dot{e}_1 + \alpha_1 Tanh(e_1) + Tanh(e_f), \tag{6}$$

$$r \triangleq \dot{e}_2 + \alpha_2 Tanh(e_2) + \alpha_3 e_2 \tag{7}$$

where $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$ denote constant positive control gains, and $e_f \in \mathbb{R}^n$ is an auxiliary signal whose dynamics are given by

$$\dot{e}_f \triangleq Cosh^2(e_f)\{-\gamma_1 e_2 + Tanh(e_1) - \gamma_2 Tanh(e_f)\} \tag{8}$$

and $\gamma_1, \gamma_2 \in \mathbb{R}$ are constant positive control gains. The auxiliary signal r is introduced to facilitate the stability analysis and is not used in the control design since the expression in (7) depends on the unmeasurable generalized state $\ddot{x}$. The structure of the error systems (and included auxiliary signals) is motivated by the need to inject and cancel terms in the subsequent stability analysis, and will become apparent in Section V.

IV. Control Development

An open-loop tracking error can be obtained by utilizing the filtered tracking error in (7) and substituting (1), (5), (6), and (8) to yield

$$r = S - f_d + \ddot{x}_d - d - u(t) - \gamma_1 e_2 \tag{9}$$

where the auxiliary function $S \in \mathbb{R}^n$ is defined as

$$\begin{aligned} S \triangleq\ & f_d - f - \gamma_2 Tanh(e_f) \\ & + \alpha_1 Cosh^{-2}(e_1)[e_2 - \alpha_1 Tanh(e_1) - Tanh(e_f)] \\ & + \alpha_2 Tanh(e_2) + \alpha_3 e_2 + Tanh(e_1), \end{aligned} \tag{10}$$

and $f_d = f(x_d, \dot{x}_d, t) \in \mathbb{R}^n$ is a desired trajectory dependent auxiliary term.

Based on the form of (9) and through an iterative stability analysis, the continuous controller, u, is designed as[4]

$$u \triangleq \gamma_1 Tanh(v) \tag{11}$$

[4]An important feature of the controller in (11) is its applicability to the case where constraints exist on the available control. Note that the control law is upper bounded by the adjustable control gain γ_1 as $\|u\| \leq \sqrt{n} \cdot \gamma_1$ where n is the dimension of u.

where $v \in \mathbb{R}^n$ is the Filippov solution to the following differential equation

$$\dot{v} = Cosh^2(v)\left[\alpha_2 Tanh(e_2) + \alpha_3 e_2 + \beta sgn(e_2) - \alpha_1 Cosh^{-2}(e_1)\, e_2 + \gamma_2 e_2\right], \quad (12)$$

where $\beta \in \mathbb{R}$ is a positive constant control gain and $sgn(\cdot)$ is defined $\forall \xi \in \mathbb{R}^m = \left[\begin{array}{cccc} \xi_1 & \xi_2 & ... & \xi_m \end{array}\right]^T$ as $sgn(\xi) \triangleq \left[\begin{array}{cccc} sgn(\xi_1) & sgn(\xi_2) & ... & sgn(\xi_m) \end{array}\right]^T$.

In review of (5)-(10), the control strategy in (11) and (12) entails several components including the development of the filtered error systems in (6) and (7), which are composed of saturated hyperbolic tangent functions designed from the Lyapunov analysis. The motivation for the design of (8) stems from the need to inject a $-\gamma_1 e_2$ signal into the closed-loop error system and to cancel terms in the analysis. Based on the stability analysis methods associated with the RISE control strategy (cf. [26], [27]), an extra derivative is applied to the closed-loop error system. The time derivative of (11) will include a $Cosh^{-2}(v)$ term. The design of (12) is motivated by the desire to cancel the $Cosh^{-2}(v)$ term, enabling the remaining terms to provide the desired feedback and cancel nonconstructive terms and disturbances as dictated by the subsequent stability analysis.

The closed-loop tracking error system can be developed by taking the time derivative of (9), and using the time derivative of (11) to yield

$$\dot{r} = \tilde{N} + N_d - \gamma_1 r - \gamma_1 \beta sgn(e_2) - Tanh(e_2) - e_2 \quad (13)$$

where $\tilde{N} \in \mathbb{R}^n$ and $N_d \in \mathbb{R}^n$ are defined as

$$\tilde{N} \triangleq \dot{S} + \gamma_1 \alpha_1 Cosh^{-2}(e_1)\, e_2 - \gamma_1 \gamma_2 e_2 + Tanh(e_2) + e_2, \quad (14)$$

$$N_d \triangleq \dddot{x}_d - \dot{f}_d - \dot{d}. \quad (15)$$

The structure of (13) is motivated by the desire to segregate terms that can be upper bounded by state-dependent terms and terms that can be upper bounded by constants. By applying the Mean Value Theorem, an upper bound can be developed for the expression in (14) as

$$\left\|\tilde{N}\right\| \leq \rho(\|w\|)\,\|w\| \quad (16)$$

where the bounding function $\rho \in \mathbb{R}$ is a positive, strictly increasing function,[5] and $w \in \mathbb{R}^{5n}$ is defined as

$$w \triangleq \left[Tanh^T(e_1),\ e_2^T,\ r^T,\ Tanh^T(e_f)\right]^T. \quad (17)$$

From Assumptions 1 and 2, the following inequality can be developed based on the expression in (15):

$$\|N_d\| \leq \zeta_{N_{d1}}, \quad \left\|\dot{N}_d\right\| \leq \zeta_{N_{d2}} \quad (18)$$

where $\zeta_{N_{d1}}, \zeta_{N_{d2}} \in \mathbb{R}$, are known positive constants.

[5]The proof in [28, App A] can be used to show that there exists a positive, nondecreasing bounding function for $\left\|\tilde{N}\right\|$. Any positive nondecreasing function can be upper bounded by a positive strictly increasing function, ρ.

V. Stability Analysis

To facilitate the stability analysis, let $\mathcal{D} \subseteq \mathbb{R}^{4n+1}$ be the open and connected set defined as $\mathcal{D} \triangleq \left\{y \in \mathbb{R}^{4n+1} \mid \|y\| \leq inf\left(\rho^{-1}\left(\left[2\sqrt{\lambda\gamma_b}, \infty\right)\right)\right)\right\}$, where λ and γ_b are subsequently defined. Let $z \in \mathbb{R}^{4n}$ be defined as

$$z \triangleq \left[e_1^T,\ e_2^T,\ r^T,\ Tanh^T(e_f)\right]^T \quad (19)$$

and $y \in \mathcal{D}$ be defined as

$$y \triangleq \left[\begin{array}{cc} z^T & \sqrt{P} \end{array}\right]^T. \quad (20)$$

In (20), the auxiliary function $P \in \mathbb{R}$ is defined as the Filippov solution to the following differential equation

$$\dot{P} \triangleq -r^T\left(N_d - \beta\gamma_1 sgn(e_2)\right), \quad (21)$$

$$P(e_2(t_0), t_0) = \beta\gamma_1 \sum_{i=1}^{n} |e_{2i}(t_0)| - e_2(t_0)^T N_d(t_0)$$

where the subscript $i = 1, 2, ..., n$ denotes the i-th element of the vector. Provided the sufficient condition for β in (24) is satisfied, $P \geq 0$ [21]. Let $V_L : \mathcal{D} \to \mathbb{R}$ be a positive-definite, locally Lipschitz, regular function defined as

$$V_L \triangleq \sum_{i=1}^{n} ln(cosh(e_{1i})) + \sum_{i=1}^{n} ln(cosh(e_{2i})) + \frac{1}{2}e_2^T e_2 + \frac{1}{2}r^T r + \frac{1}{2}Tanh^T(e_f)\, Tanh(e_f) + P \quad (22)$$

where e_{1i} and e_{2i} denote the i-th element of the vector e_1 and e_2, respectively. The Lyapunov function candidate in (22) satisfies the following inequalities:

$$\phi_1(y) \leq V_L(y) \leq \phi_2(y), \quad \forall y \in \mathcal{D}. \quad (23)$$

Based on (4) and (22), the continuous, positive definite, strictly increasing functions $\phi_1, \phi_2 : \mathcal{D} \to \mathbb{R}$ in (23) are defined as $\phi_1 \triangleq \frac{1}{2}tanh^2(\|y\|)$, $\phi_2 \triangleq \frac{3}{2}\|y\|^2$. Additionally, let $\mathcal{S} \subset \mathcal{D}$ denote a set defined as $\mathcal{S} \triangleq \left\{y \in \mathcal{D} \mid \rho\left(\sqrt{\phi_2(y)}\right) < 2\sqrt{\lambda\gamma_b}\right\}$.

Theorem 1. *Given the dynamics in (1), the controller given by (11) and (12) ensures local*[6] *asymptotic tracking in the sense that all Filippov solutions, such that $y(t_0) \in \mathcal{S}$, are bounded, and satisfy*

$$\|e_1\| \to 0 \quad as \quad t \to \infty$$

provided the control gains are selected sufficiently large based on the initial conditions of the states and the following sufficient conditions

$$\alpha_1 > \frac{1}{2}, \quad \alpha_2 > 0, \quad \alpha_3 > \frac{1}{2} + \frac{\gamma_1^2\zeta^2}{4}, \quad \gamma_2 > \frac{1}{\zeta^2},$$

[6]For arbitrarily large initial conditions or arbitrarily large disturbances, the control gains required to satisfy the sufficient gain conditions in (24) may demand an input that is not physically deliverable by the system (i.e., the gain γ_1 may be required to be larger than the saturation limit of the actuator). Despite gain dependency on the system's initial conditions, this result does not satisfy the standard semi-global result because, under the consideration of input constraints, γ_b cannot be arbitrarily increased and consequently the region of attraction cannot be arbitrarily enlarged to include all initial conditions. This outcome is not surprising from a physical perspective in the sense that such demands may yield cases where the actuation is insufficient to stabilize the system.

$$\beta\gamma_1 > \zeta_{N_{d1}} + \frac{\zeta_{N_{d2}}}{\alpha_3}, \qquad 4\lambda\gamma_b > \rho^2\left(\|w(0)\|\right) \tag{24}$$

where $\lambda \in \mathbb{R}^+$ *is defined as* $\lambda = min\left\{\alpha_1 - \frac{1}{2},\ 2\alpha_2 + \alpha_3,\ \alpha_3 - \frac{1}{2} - \frac{\gamma_1^2\zeta^2}{4},\ \gamma_2 - \frac{1}{\zeta^2},\ \gamma_a\right\}$, $\|w(0)\|$ *denotes the initial conditions of the state, and* $\gamma_b, \zeta \in \mathbb{R}$ *are subsequently defined adjustable positive constants.*

Proof: Let $y(t)$ for $t \in [t_0, \infty)$ denote a Filippov solution to the differential equation $\dot{y} = h_3(y,t)$ such that $y(t_0) \in \mathcal{S}$, where $h : \mathbb{R}^{4n+1} \times \mathbb{R}_{\geq 0} \to \mathbb{R}^{4n+1}$ denotes the right hand side of the closed-loop error signals. Using Filippov's theory of differential inclusions [29], [30], the existence of solutions can be established for $\dot{y} \in K[h](y,t)$, where $K[h](y,t) \triangleq \bigcap_{\delta>0} \bigcap_{\mu S_m = 0} \overline{co}h\left(B(y,\delta) \backslash S_m, t\right)$, where $\bigcap_{\mu S_m = 0}$ denotes the intersection of all sets S_m of Lebesgue measure zero, $\overline{co}$ denotes convex closure, and $B(y,\delta) = \{\varsigma \in \mathbb{R}^n \mid \|y - \varsigma\| < \delta\}$ [31], [32]. The time derivative of (22) along the Filippov trajectories exists almost everywhere (a.e.), i.e., for almost all $t \in [t_0, t_f]$, and $\dot{V}_L \overset{a.e.}{\in} \dot{\tilde{V}}_L$ where

$$\dot{\tilde{V}}_L = \bigcap_{\xi \in \partial V_L(y,t)} \xi^T K\left[\dot{e}_1^T\ \dot{e}_2^T\ \dot{r}^T\ Cosh^{-2}(e_f)\,\dot{e}_f^T\ \frac{1}{2}P^{-\frac{1}{2}}\dot{P}\ 1\right]^T,$$

and ∂V_L is the generalized gradient of V_L [33]. Since V_L is a continuously differentiable

$$\dot{\tilde{V}} \subset \nabla V_L^T K\left[\dot{e}_1^T\ \dot{e}_2^T\ \dot{r}^T\ Cosh^{-2}(e_f)\,\dot{e}_f^T\ \frac{1}{2}P^{-\frac{1}{2}}\dot{P}\right]^T \tag{25}$$

where

$$\nabla V_L \triangleq \left[Tanh^T(e_1), \left(Tanh^T(e_2) + e_2^T\right), r^T, Tanh^T(e_f), 2P^{\frac{1}{2}}\right]^T.$$

Using the calculus for $K[\cdot]$ from [32], and substituting (5)-(8), and (13) into (25), yields

$$\begin{aligned}\dot{\tilde{V}}_L \subset\ & r^T\left(\tilde{N} + N_d - \gamma_1 r - Tanh(e_2) - e_2\right)\\ & - r^T\left(\gamma_1\beta K\left[sgn(e_2)\right]\right)\\ & + Tanh^T(e_1)\left(e_2 - \alpha_1 Tanh(e_1) - Tanh(e_f)\right)\\ & + Tanh^T(e_2)\left(r - \alpha_2 Tanh(e_2) - \alpha_3 e_2\right)\\ & + e_2^T\left(r - \alpha_2 Tanh(e_2) - \alpha_3 e_2\right)\\ & + Tanh^T(e_f)\left(-\gamma_1 e_2 + Tanh(e_1)\right)\\ & + Tanh^T(e_f)\left(-\gamma_2 Tanh(e_f)\right) + \dot{P}\end{aligned} \tag{26}$$

where $K[sgn(e_2)] = SGN(e_2)$ [32]. Substituting (21), (7), (16), and (18), the expression in (26) reduces to the scalar inequality

$$\begin{aligned}\dot{V}_L \overset{a.e.}{\leq}\ & -\alpha_1\|Tanh(e_1)\|^2 - (2\alpha_2 + \alpha_3)\|Tanh(e_2)\|^2\\ & -\alpha_3\|e_2\|^2 - \gamma_2\|Tanh(e_f)\|^2 - \gamma_1\|r\|^2\\ & +\rho\|w\|\|r\| + \|Tanh(e_1)\|\|e_2\|\\ & +\gamma_1\|Tanh(e_f)\|\|e_2\|\end{aligned} \tag{27}$$

where the set in (26) reduces to the scalar equality in (27) since the RHS is continuous a.e., i.e, the RHS is continuous except for the Lebesgue negligible set of times when $r^T\gamma_1\beta K[sgn(e_2)] - r^T\gamma_1\beta K[sgn(e_2)] \neq 0$ [31], [34].[7] Young's Inequality can be applied to select terms in (27) as

$$\begin{aligned}\|Tanh(e_1)\|\|e_2\| &\leq \frac{1}{2}\|Tanh(e_1)\|^2 + \frac{1}{2}\|e_2\|^2 \\ \gamma_1\|Tanh(e_f)\|\|e_2\| &\leq \frac{1}{\zeta^2}\|Tanh(e_f)\|^2 + \frac{\gamma_1^2\zeta^2}{4}\|e_2\|^2.\end{aligned} \tag{28}$$

To facilitate the subsequent stability analysis, let γ_1 be selected as $\gamma_1 = \gamma_a + \gamma_b$, where $\gamma_a, \gamma_b \in \mathbb{R}$ are positive gain constants. Utilizing (28), completing the squares on r and grouping terms, the expression in (27) can be upper bounded by

$$\begin{aligned}\dot{V}_L \overset{a.e.}{\leq}\ & -\left(\alpha_1 - \frac{1}{2}\right)\|Tanh(e_1)\|^2 - (2\alpha_2 + \alpha_3)\|Tanh(e_2)\|^2\\ & -\left(\alpha_3 - \frac{1}{2} - \frac{\gamma_1^2\zeta^2}{4}\right)\|e_2\|^2 - \left(\gamma_2 - \frac{1}{\zeta^2}\right)\|Tanh(e_f)\|^2\\ & -\gamma_a\|r\|^2 + \frac{\rho^2(\|w\|)\|w\|^2}{4\gamma_b}.\end{aligned} \tag{29}$$

Provided the sufficient conditions in (24) are satisfied, (17) and (19) can be used to conclude that

$$\dot{V}_L \overset{a.e.}{\leq} -\phi_3(\|z\|) \leq -U(y) \tag{30}$$

where $\phi_3 \in \mathbb{R}$ is defined as $\phi_3 \triangleq \left(\lambda - \frac{\rho^2(\|w\|)}{4\gamma_b}\right)\tanh^2(\|z\|)$, λ was defined in (24), and $U \triangleq c\tanh^2(\|z\|)\ \ \forall y \subset \mathcal{D}$ is a continuous, positive semi-definite function for some positive constant $c \in \mathbb{R}$.

The inequalities in (23) and (30) can be used to show that $V_L \in \mathcal{L}_\infty$ in $\mathcal{D}$, hence, $e_1, e_2, r, Tanh(e_f) \in \mathcal{L}_\infty$ in $\mathcal{D}$. From (2), $Tanh(e_1), Tanh(e_2) \in \mathcal{L}_\infty$ and from (6) and (7), $\dot{e}_1, \dot{e}_2 \in \mathcal{L}_\infty$ in $\mathcal{D}$. From (11), $u \in \mathcal{L}_\infty$. From Assumption 2 and by utilizing the fact that $e_1, \dot{e}_1 \in \mathcal{L}_\infty$, $q, \dot{q} \in \mathcal{L}_\infty$ in $\mathcal{D}$. From (17), $w \in \mathcal{L}_\infty$ in $\mathcal{D}$. Assumption 1, (13), (16) and (18) can be used to show that $\dot{r} \in \mathcal{L}_\infty$ in $\mathcal{D}$. Utilizing (8) and the fact that $e_2 \in \mathcal{L}_\infty$ in $\mathcal{D}$, the product $Cosh^{-2}(e_f)\,\dot{e}_f \in \mathcal{L}_\infty$ in $\mathcal{D}$. Thus, $\dot{z} \in \mathcal{L}_\infty$ in $\mathcal{D}$, and it can be shown that z is uniformly continuous (UC) in $\mathcal{D}$. Since z is UC, $tanh(\|z\|)$ is UC. The definitions of U and z can be used to prove that U is UC in $\mathcal{D}$.

Since ρ and ϕ_2 are strictly increasing functions, the region of attraction can be increased by increasing the gains. The fact that $\rho(0) = c \neq 0$ for some $c \in \mathbb{R}_+$ implies that in order to obtain a non-trivial region of attraction, the saturation bound γ has to be large enough so that $2\sqrt{\lambda\gamma_b} > c$. From (30), [36, Corollary 1] can be invoked to show that $tanh(\|z\|) \to 0\ \ as\ \ t \to \infty\ \forall y(0) \in \mathcal{S}$. Based on the definition of z in (19), $\|e_1\| \to 0\ \ as\ \ t \to \infty\ \forall y(0) \in \mathcal{S}$. ■

[7] The set of times $\Lambda \triangleq \left\{t \in [0,\infty) : r(t)^T\gamma_1\beta K[sgn(e_2(t))] - r(t)^T\gamma_1\beta K[sgn(e_2(t))] \neq 0 \subset [0,\infty)\right.$ is equivalent to the set of times $\{t : e_2(t) = 0 \wedge r(t) \neq 0\}$. From (7), this set can also be represented by $\{t : e_2(t) = 0 \wedge \dot{e}_2(t) \neq 0\}$. Provided $e_2(t)$ is continuously differentiable, it can be shown that the set of time instances $\{t : e_2(t) = 0 \wedge \dot{e}_2(t) \neq 0\}$ is isolated, and thus, measure zero. This implies that the set Λ is measure zero. [35]

VI. Experimental Results

To examine the performance of the saturated RISE approach, the controller in (11) and (12) was implemented on a planar manipulator testbed.[8] The manipulator can be modeled as an Euler-Lagrange system with the following dynamics

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + F(\dot{q}) = \tau \tag{31}$$

where $M \in \mathbb{R}^{2\times 2}$ denotes the inertia matrix, $V_m \in \mathbb{R}^{2\times 2}$ denotes the unknown centripetal-Coriolis matrix, $F \in \mathbb{R}^2$ denotes a continuously differentiable friction model given in [24] as $F \triangleq M^{-1}(q)(\gamma_1(\tanh(\gamma_2\dot{q}) - \tanh(\gamma_3\dot{q})) + \gamma_4 \tanh(\gamma_5\dot{q}) + \gamma_6\dot{q})$ where $\gamma_i\ \forall i = 1,2,...,6$ are unknown positive constants that are related to friction coefficients. Additionally, in (31), $q, \dot{q}, \ddot{q} \in \mathbb{R}^2$ denote the link position, velocity and acceleration, and $\tau \in \mathbb{R}^2$ denotes the control torque. The Euler-Lagrange dynamics can be rewritten as

$$\ddot{x} = f(x,\dot{x},t) + \chi(x,\dot{x},t) + u(x,t) + d(t) \tag{32}$$

where $x = \begin{bmatrix} q_1(t), & q_2(t) \end{bmatrix}^T$, $f \triangleq -M^{-1}(q)V_m(q,\dot{q})\dot{q}$, $u \triangleq M^{-1}(q)\tau(t)$, $\chi \triangleq M^{-1}(q)F(\dot{q}) - M^{-1}(q_d)F(\dot{q}_d)$ and $d \triangleq M^{-1}(q_d)F(\dot{q}_d)$. From (32) and [24], the friction disturbance d satisfies Assumption 1. Other disturbances such as wind, ocean currents, etc. can also be shown to satisfy Assumption 1 for other electromechanical and aerospace systems. Moreover, given known bounds on the desired trajectory, and conservative upper bound estimates for the constant inertial parameters and friction constants, the inequalities in (24) can also be satisfied. The auxiliary term $\chi(x,\dot{x},t)$ can be combined with $f(x,\dot{x},t)$-like terms in $\tilde{N}(\cdot)$.

The control objective is to track a desired link trajectory, selected as $q_{d1} = q_{d2} = (45 + 60 sin(2t))\left(1 - e^{-0.01t^3}\right) deg$. The initial conditions for the manipulator were selected a complete rotation away from the initial conditions of the desired trajectory as $q_1(0) = 360\ deg$ and $q_2(0) = -180\ deg$. The control torque was arbitrarily selected to be artificially limited (well-within the capabilities of the actuator) to $|\tau_1| \leq 60\ Nm$, $|\tau_2| \leq 15\ Nm$. Specifically, the feedback gains for the proposed controller were selected as $\gamma_1 = diag(52, 13)$, $\gamma_2 = diag(22, 19)$, $\beta = diag(3.8, 3.8)$, $\alpha_1 = diag(6.2, 6.0)$, $\alpha_2 = diag(8, 11)$, $\alpha_3 = diag(45, 45)$, $e_f(0,0)$ is selected as $e_f(0,0) = 0$ and $v(0,0)$ is selected such that $u(0) = 0$.[9]

[8]The manipulator consists of a two-link direct drive revolute robot consisting of two aluminum links, mounted on 240.0 N-m (base joint) and 20.0 N-m (second joint) switched reluctance motors. The motor resolvers provide rotor position measurements with a resolution of 614,400 pulses/revolution, and a standard backwards difference algorithm is used to numerically determine velocity from the encoder readings. Data acquisition and control implementation were performed in real-time using QNX at a frequency of 1.0 kHz.

[9]It is important to note that for the given Euler-Lagrange system, the implemented controller is $\tau = M(q)u$. Thus, the bound on the implemented control will include the (known) bound on the inertia matrix. For this experiment, the inertia matrix can be bounded by $\|M(q)\| \leq 1.15$.

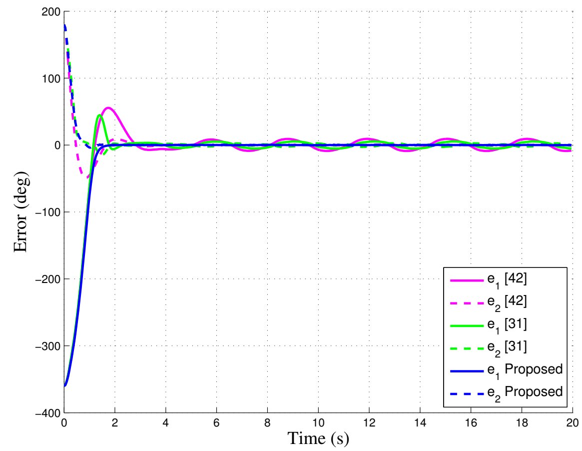

Figure 1. Tracking errors versus time for a) classical PID with integral clamping anti-windup, b) adaptive full-state feedback controller, and c) the proposed saturated RISE controller.

The performance of the saturated RISE control design was compared against two controllers available in literature: a classical PID controller with conditional integral clamping anti-windup [37] and an adaptive full-state feedback controller with bounded inputs [25]. Each controller was tuned to achieve the best possible performance, given the saturation bounds. Since each controller has a different structure, it is difficult to comment on the comparative nature of the gains which were implemented. Starting with the same large initial condition offset, the tracking errors of each controller are depicted in Figure 1. The control torques for each controller are shown in Figure 2 and each remain within the prescribed bounds.

The results show that the PID w/ AW [37] achieves steady-state RMS errors of 8.3857 deg and 1.4096 deg for each joint, respectively. The steady-state RMS torque for each joint was found to be 14.5630 Nm and 1.8732 Nm, respectively. The Adaptive FSFB [25] steady-state RMS errors and torques for each joint were 3.8232 deg, 1.6537 deg, 14.9367 Nm and 1.2360 Nm, respectively. The proposed Saturated RISE steady-state RMS errors and torques for each joint were 0.1607 deg, 0.2889 deg, 14.3363 Nm and 1.1883 Nm, respectively. The results illustrates that for comparable RMS torque values, the saturated RISE controller exhibits improved steady-state performance when compared to the other control designs.

VII. Conclusion

A continuous saturated controller is developed for a class of uncertain nonlinear systems which includes time-varying and nonlinearly parameterized functions and additive bounded disturbances. The bound on the control is known a priori and can be adjusted by changing the feedback gains. The saturated controller is shown to guarantee local asymptotic tracking despite uncertainty in the dynamics using smooth hyperbolic functions. Experimental results using a two-link robot manipulator demonstrate the performance of the proposed controller.

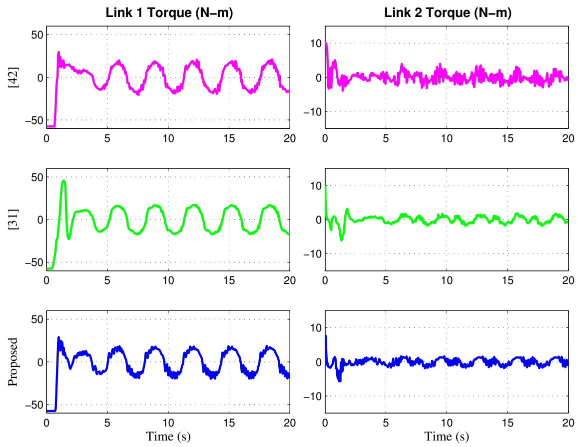

Figure 2. Control torques versus time for a) classical PID with integral clamping anti-windup, b) adaptive full-state feedback controller, and c) the proposed saturated RISE controller. Despite starting an entire revolution away from the desired trajectory, the controller only saturates at the preset artificial saturation limits briefly.

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