Adaptive RISE Feedback Control Strategies for Systems with Structured and Unstructured Uncertainties

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This paper presents an overview of adaptive control strategies for systems with structured and unstructured uncertainties via the continuous Robust Integral of the Sign of the Error (RISE) feedback. The RISE control strategy has been shown to yield asymptotic results in the presence of modeling uncertainties and exogenous disturbances. Combining the RISE feedback control structure with feedforward adaptive techniques has the potential to improve transient performance and reduce control effort while maintaining the asymptotic result.

I. Introduction

THE development of controllers capable of compensating for uncertain nonlinear systems remains a mainstream research area. A variety of adaptive feedforward components have been developed to yield asymptotic results for systems with linearly parametrized (LP) uncertainties.[1–3] Some results have also focused on applying adaptive controllers such as neural network (NN)[4–10] and fuzzy logic[11–13] controllers to systems that have unstructured uncertainties (i.e., the uncertainty does not satisfy the LP assumption). Asymptotic tracking for uncertain nonlinear systems with unstructured uncertainties is traditionally achieved using either high gain feedback or discontinuous sliding modes in conjunction with adaptive feedforward terms. In the case of high gain controllers, asymptotic tracking is only achieved as the gain goes to infinity, and discontinuous sliding mode techniques require infinite bandwidth.

The control strategy, referred to as Robust Integral of the Sign of the Error (RISE) in Ref. 14, was developed to compensate for sufficiently smooth unstructured uncertainties. The RISE architecture enables the design of continuous controllers that achieve asymptotic tracking with finite gains.[14–19] RISE results are achieved by filtering the sliding mode through an integrator to generate a locally absolutely continuous control signal. Asymptotic convergence of the tracking error is achieved through the combination of a gradient adaptive update law and RISE feedback for a nonlinear system with LP uncertainties.[14]

When combined with a gradient adaptive update law, the RISE feedback term guarantees[14] asymptotic convergence of the tracking error in the presence of linear-in-the-parameter (LP) modeling uncertainties and exogenous disturbances; however, despite the convergence of the rate of the adaptive update, convergence of the parameter estimate error is not guaranteed. This motivates the addition of a parameter estimate error into the adaptive update law, which is problematic since it is unknown. Although adaptive update laws exist that include a measurable form of the prediction error, no stability result has been developed for systems with exogenous disturbances. Furthermore, it is not clear how the so-called swapping approach[1] (also described as input filtering) can be applied to systems with non-LP disturbances because the unknown disturbance terms get filtered, are included in the filtered control input, and will cause non-LP disturbances to appear in the prediction error. Semi-global asymptotic tracking can be achieved through the use of a RISE structure in the prediction error as well as the control input.[15]

Lyapunov-based stability methods have been used extensively to design the controller and adaptive update laws simultaneously for nonlinear systems with LP uncertainties; however, this approach restricts the design of the update laws since they are required to cancel cross terms in the stability analysis. Since the RISE feedback alone can yield asymptotic results in the presence of structured and unstructured uncertainties, it can be used in concert with an adaptive update law that is designed solely for improving transient performance and/or reducing control effort. A modular adaptive RISE control strategy that yields asymptotic tracking with a generic form for the adaptive update law and parameter estimate is developed in Ref. 16. In Ref. 14–16, the RISE structure is used to damp out the additive

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disturbances and some bounded cross terms resulting from the adaptation laws. This ability of the RISE structure to counter bounded disturbances can be exploited to compensate for non-LP uncertainties, where the non-LP plant dynamics can be approximated only up to a bounded approximation error.

NNs gained popularity as a feedforward method for compensating for non-LP uncertainties. Due to the residual function reconstruction error associated with NN approximations, typical NN-based controllers yield uniformly ultimately bounded (UUB) stability results. Asymptotic results have been obtained via the addition of a discontinuous feedback term; however, chattering and infinite control bandwidth limit the utility of these controllers. The addition of a RISE feedback control structure to a NN-based controller ensures asymptotic tracking with a continuous controller.

Ref. 17 achieved asymptotic tracking in the presence of non-LP uncertainties via the combination of a continuous RISE feedback term and a NN feedforward term. Combining the NN and RISE methods raises several technical challenges. First, the NN has to be constructed in terms of a desired trajectory to avoid adaptive update laws that are dependent on acceleration terms. The manner in which the NN weight estimates appear in the Lyapunov derivative causes problems in stability analysis methods developed for previous RISE-based controllers. Previously, terms in the stability analysis that are upper bounded by constants must have time derivatives that can also be upper bounded by a constant. Due to a smooth projection algorithm, the norm of the NN weight estimates can be upper bounded by a constant; however, the time derivatives are state-dependent. This issue is addressed via a modified RISE stability analysis that results in altered (but not more restrictive) sufficient gain conditions.[17]

The RISE technique enables the development of adaptive controllers that yield asymptotic results in the presence of non-LP uncertainties and exogenous disturbances. Furthermore, as described in Section II.C, the RISE architecture is flexible in the sense that it facilitates the design of innovative adaptive update laws while maintaining asymptotic properties of the system. This flexibility, along with the robustness property of the RISE-based design, can be exploited further to design asymptotic adaptive controllers with improved performance. For example, based on ideas from reinforcement learning, an actor-critic (AC) based adaptive update scheme is developed in Ref. 20 for improved performance.

The remainder of the paper is organized as follows: Section II briefly describes the application of the RISE technique to a nonlinear system with LP uncertainties, and Section III discusses the combination of a NN feedforward term with the RISE structure to compensate for non-LP uncertainties. The level of detail in each section is condensed to highlight the features, advantages and nuances in the application of the RISE control structure. For a comprehensive treatment, the reader is referred to the references contained within each section.

II. RISE-based Control for Systems With Structured Uncertainties

II.A. RISE-based control with gradient adaptation law[14]

For illustrative purposes, consider a dynamic system described by second order Euler-Lagrange dynamics as

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) + F(q) + h = u, \tag{1}$$

where $q, \dot{q}, \ddot{q} \in \mathbb{R}^n$ are the generalized position, velocity, and acceleration vectors, $M : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ denotes a generalized positive-definite inertia matrix, $V_m : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$ denotes a generalized centripetal-Coriolis matrix, $G : \mathbb{R}^n \to \mathbb{R}^n$ denotes a generalized gravity vector, $F : \mathbb{R}^n \to \mathbb{R}^n$ denotes a generalized friction vector, $h \in \mathbb{R}^n$ denotes a generalized disturbance, and $u \in \mathbb{R}^n$ denotes the input. The disturbance h and its first two time derivatives are assumed to be uniformly bounded by known positive constants, and the uncertain matrices M and V_m and the vectors G and F are assumed to be LP. The control objective is to track a desired trajectory $q_d \in \mathbb{R}^n$.

To facilitate the RISE-based control development, three filtered tracking errors can be defined as

$$\begin{aligned} e_1 &\triangleq q_d - q, \\ e_2 &\triangleq \dot{e}_1 + \alpha_1 e_1, \\ r &\triangleq \dot{e}_2 + \alpha_2 e_2, \end{aligned} \tag{2}$$

where $\alpha_1, \alpha_2 \in \mathbb{R}$ are positive constants. The filtered tracking error r can be expressed as

$$M(q)r = Y_d\theta + S + h - u, \tag{3}$$

where

$$Y_d\theta = M(q_d)\ddot{q}_d + V_m(q_d,\dot{q}_d)\dot{q}_d + G(q_d) + F(\dot{q}_d). \tag{4}$$

In Eq. (4), $\theta \in \mathbb{R}^p$ denotes the vector of constant unknown parameters, $Y_d \in \mathbb{R}^{n\times p}$ is the regression matrix composed of q_d, $\dot{q}_d$ and $\ddot{q}_d$, and S contains all the remaining terms. In traditional sliding mode design, the disturbance h is damped out by including an infinite frequency discontinuous term in u. Driven by the desire to obtain a continuous controller, the RISE architecture moves the sliding mode to the derivative of the control signal u. The time-derivative of the control signal appears in the time-derivative of the filtered tracking error r. Therefore, the RISE analysis involves an extra derivative of the states. The controller

$$u = -Y_d\hat{\theta} + \mu, \tag{5}$$

is comprised of an adaptive feedforward component $Y_d\hat{\theta}$ and a RISE-based feedback component μ, where $\hat{\theta} \in \mathbb{R}^p$ is the estimate of the vector θ.

Using Eq. (5), the time derivative of the filtered tracking error r can be written as

$$M(q)\dot{r} = -\frac{1}{2}\dot{M}(q)r + \dot{Y}_d\tilde{\theta} + \tilde{N} + N_d - \dot{\mu} - e_2, \tag{6}$$

where $\tilde{\theta} = \theta - \hat{\theta}$. In Eq. (6), $\tilde{N}$ contains all the terms that are linear in e_1, e_2 and r, and all the terms that can be written as $g(q,\dot{q}) - g(q_d,\dot{q}_d)$, where $g : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is a smooth function. The motivation behind this segregation is that the Mean Value Theorem can be used to obtain a bound on $\tilde{N}$ as

$$\left\|\tilde{N}\right\| \leq \rho(\|z\|)\|z\|, \tag{7}$$

where $z \triangleq \left[e_1^T, e_2^T, r^T\right] \in \mathbb{R}^{3n}$ and $\rho : \mathbb{R} \to \mathbb{R}$ is a positive, non-decreasing function. The term N_d in Eq. (6) contains all the terms that are functions of the desired trajectory, the disturbance, and their derivatives. Since the desired trajectory and the disturbance are assumed to be bounded and sufficiently smooth, N_d and $\dot{N}_d$ can be bounded as

$$\|N_d\| \leq \zeta_{N_d}, \quad \left\|\dot{N}_d\right\| \leq \zeta_{N_{d2}}, \tag{8}$$

where ζ_{N_d} and $\zeta_{N_{d2}}$ are known positive constants.

Given the form of Eq. (6), and the fact that N_d is bounded by a known constant, traditional sliding mode design would suggest that a term of the form $-\zeta_{N_d} sgn(r)$ should be added to $\dot{\mu}$ to compensate for N_d. However, r contains the generalized acceleration $\ddot{q}$, and hence, is not measurable. Thus, the control design implements the sliding mode using the measurable error signal e_2 as[14, 18, 19]

$$\mu = (k_s+1)(e_2 - e_2(0)) + \nu, \tag{9}$$

where $\nu \in \mathbb{R}^n$ is a Filippov solution to the differential equation

$$\dot{\nu} = (k_s+1)\alpha_2 e_2 + \beta sgn(e_2).$$

The parameters $\hat{\theta}$ are updated using a standard gradient descent update law as

$$\dot{\hat{\theta}} = \Gamma Y_d^T r. \tag{10}$$

Since Y_d is only a function of desired trajectories, the update law in Eq. (10) is implementable using integration by parts as

$$\hat{\theta} = \hat{\theta}(0) + \Gamma\dot{Y}_d^T(\sigma)e_2(\sigma)\Big|_0^t - \Gamma\int_0^t \left(\ddot{Y}_d^T(\sigma)e_2(\sigma) - \alpha_2\dot{Y}_d^T(\sigma)e_2(\sigma)\right)d\sigma.$$

Asymptotic tracking of the desired trajectory can be established for the proposed controller via a Lyapunov analysis.[14] Consider a standard quadratic Lyapunov candidate function

$$V_0 = \frac{1}{2}r^T M r + e_1^T e_1 + \frac{1}{2}e_2^T e_2 + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta}. \tag{11}$$

Since the controller in Eq. (9) uses $sgn(e_2)$, direct cancellation of N_d in the Lyapunov derivative becomes infeasible. Instead, the Lyapunov derivative contains the term $r^T(N_d - \beta sgn(e_2))$, which highlights where the disturbance enters the analysis; however, it is unclear at this stage how the $-\beta sgn(e_2)$ term injected through the control is beneficial. Motivated by this issue, a non-quadratic Lyapunov candidate can be used where Eq. (11) is modified to include the auxiliary term P[14,18] defined as a Filippov solution to the differential equation $\dot{P} = -r^T(N_d - \beta sgn(e_2))$, with the initial condition $P(0) = \beta\sum_{i=1}^{n}|e_{2i}(0)| - e_2^T(0)N_d(0)$ is added to the Lyapunov function, which injects the desired $-r^T(N_d - \beta sgn(e_2))$ term in the Lyapunov derivative. For $V_0 + P$ to be a Lyapunov function, the auxiliary term P needs to be positive. The control term $\beta sgn(e_2)$ plays a role towards this end. Specifically, under the gain condition $\beta > \zeta_{N_d} + \frac{1}{\alpha_2}\zeta_{N_{d2}}$, $P \geq 0$, and under the further gain conditions $\alpha_1 > \frac{1}{2}$ and $\alpha_2 > 1$, semi-global asymptotic tracking of the desired trajectory is achieved.[14,18]

In the presented RISE structure, if accurate estimates of the parameters are available, the feedforward component of the controller can be made to match the plant dynamics, resulting in better cancellation of the nonlinearities. This heuristic motivates the need to drive the parameter updates based on the parameter estimation error. Since the estimation error is unknown, a prediction error[15] is motivated. Although adaptive update laws exist that include a measurable form of the prediction error, results prior to Ref. 15 had not been developed for systems with exogenous disturbances. The following section presents a RISE-like filter structure in conjunction with the RISE controller to obtain asymptotic tracking for systems employing prediction error-based adaptive update laws.

II.B. RISE-based control using a composite adaptation law[15]

To illustrate the composite adaptive control result, consider a second-order Euler-Lagrange-like system as

$$\ddot{q} = f(q,\dot{q}) + G(q,\dot{q})u + h, \tag{12}$$

where $f : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ and $G : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$ are unknown nonlinear functions. The subsequent development is based on the assumptions that the matrix $G(q,\dot{q})$ is invertible for all $q, \dot{q} \in \mathbb{R}^n$ and q and $\dot{q}$ are measurable outputs. A measurable prediction error $\varepsilon \in \mathbb{R}^n$ is defined as[15]

$$\varepsilon \triangleq u_f - \hat{u}_f, \tag{13}$$

where $\hat{u}_f \in \mathbb{R}^n$ is an estimate of the filtered input $u_f \in \mathbb{R}^n$ which is computed by

$$\dot{u}_f + \omega u_f = \omega u. \tag{14}$$

In Eq. (14), $\omega \in \mathbb{R}$ is a known positive constant. Directly solving the differential equation in Eq. (14) yields

$$u_f = \upsilon * u, \quad \upsilon \triangleq \omega e^{-\omega t}, \tag{15}$$

where $*$ denotes the convolution operator. Using Eqs. (12) and the adaptive term from Eq. (4), the expression in Eq. (15) can be rewritten as

$$u_f = Y_{df}\theta + \upsilon * S - \upsilon * S_d + h_f, \tag{16}$$

where the auxiliary terms S, $S_d \in \mathbb{R}^n$ are defined as

$$S \triangleq G^{-1}\{\ddot{q} - f - h\}, \tag{17}$$

$$S_d \triangleq G_d^{-1}\{\ddot{q}_d - f_d - h\}, \tag{18}$$

the filtered regression matrix, $Y_{df} \in \mathbb{R}^{n\times p}$ is defined as $Y_{df} \triangleq \upsilon * Y_d$, and the disturbance $h_f \in \mathbb{R}^n$ is defined as $h_f \triangleq -\upsilon * g_d^{-1}h$. The non-LP disturbance term h from the system dynamics gets filtered and is included in the filtered control input. To compensate for the filtered disturbance, a RISE-like structure is included in the estimated filtered control input. Using Eq. (16), the prediction error can be expressed as

$$\varepsilon = Y_{df}\theta + \upsilon * S - \upsilon * S_d + h_f - \hat{u}_f. \tag{19}$$

The filtered estimate of the control input is designed as[15]

$$\hat{u}_f = Y_{df}\hat{\theta} + \mu_2, \tag{20}$$

where $\mu_2 \in \mathbb{R}^n$ is defined as the Filippov solution to the differential equation

$$\dot{\mu}_2 \triangleq k_2\varepsilon + \beta_2 sgn(\varepsilon).$$

The composite adaptive update law for the adaptive estimates $\hat{\theta} \in \mathbb{R}^p$ is the combination of a tracking error-based term and a prediction error term as

$$\dot{\hat{\theta}} = \Gamma \dot{Y}_d^T r + \Gamma \dot{Y}_{df}^T \varepsilon, \tag{21}$$

where $\Gamma \in \mathbb{R}^{p \times p}$ is a positive definite constant gain matrix. The combination of the controller in Eq. (5), the estimated filtered control input in Eq. (20), and the adaptive update law in Eq. (21) results in a negative semi-definite Lyapunov function derivative containing $-\varepsilon^2$ terms, from which it can be concluded that $\|\varepsilon\| \to 0$ as $t \to \infty$, in addition to $\|e_1\| \to 0$ as $t \to \infty$.[15] Experiments on a rotating disk with externally applied friction in Ref. 15 show that the composite adaptive update law provides better tracking performance compared to the typical RISE controller with gradient update laws. Three control strategies were compared: RISE without any adaptive feedforward terms, RISE with a gradient descent adaptive update law, and RISE with the developed composite update law. The average RMS tracking error for RISE with composite update law was 0.102 deg, compared to 0.219 deg and 0.138 deg for RISE alone and RISE with a gradient update law, respectively. Despite the improved tracking performance of the composite adaptive controller, no conclusion can be made regarding convergence of the parameter estimation error due to the filtered additive disturbances.

II.C. RISE-based control using modular adaptation law[16]

Consider the Euler-Lagrange dynamics previously defined in Eq. (1), the linear parametrization in Eq. (4), and the filtered tracking errors defined in Eq. (2). This section considers a framework that adds modularity to adaptive control design, facilitating the design of novel update laws. Modularity is achieved through analysis by showing that asymptotic tracking is achieved with any update law that satisfies some general conditions. The modifications to the RISE analysis necessary to establish modularity are summarized in this section. For a formal treatment, the reader is referred to Ref. 16.

To eliminate the dependence of the update laws on the system stability, the parameter estimation error-related terms are removed from the Lyapunov function, and the terms related to the update laws are handled differently in the Lyapunov derivative. Using the definition of the controller from Eqs. (5) and (9), the derivative of the filtered tracking error in Eq. (6) can be expressed as

$$M(q)\dot{r} = -\frac{1}{2}\dot{M}(q)r + \tilde{N} + N_B - (k_s+1)r - \beta_1 sgn(e_2) - e_2. \tag{22}$$

In Eq. (22), the unmeasurable/unknown auxiliary terms $\tilde{N}, N_B \in \mathbb{R}^n$ are defined as $\tilde{N} \triangleq -\frac{1}{2}\dot{M}(q)r + \dot{S} + e_2 + \tilde{N}_0$ and $N_B \triangleq N_{B_1} + N_{B_2}$, where N_{B_1} is given by $N_{B_1} \triangleq \dot{Y}_d\theta + \dot{h}$, and the sum of the auxiliary terms $\tilde{N}_0$ and N_{B_2} is given by $N_{B_2} + \tilde{N}_0 = -\dot{Y}_d\hat{\theta} - Y_d\dot{\hat{\theta}}$.

Specific definitions for $\tilde{N}_0$ and N_{B_2} are provided subsequently based on the definition of the adaptive update law for $\hat{\theta}$. The structure of Eq. (22) and the introduction of the auxiliary terms is motivated by the desire to segregate terms that can be upper bounded by state-dependent terms and terms that can be upper bounded by constants. Specifically, depending on how the adaptive update law is designed, analysis is provided in the next section to upper bound $\tilde{N}$ by state-dependent terms and upper bound N_B by a constant. The need to further segregate N_B is that some terms in N_B have time derivatives that are upper bounded by a constant, while other terms have time-derivatives that are upper-bounded by state dependent terms. The segregation of these terms based on the structure of the adaptive update law is key for the development of stability analysis for the modular RISE-based adaptive update law/controller.

The control development requires some general bounds on the structure of the adaptive update law. The subsequent development is based on the assumption that the parameter estimate $\hat{\theta}$ and the update law $\dot{\hat{\theta}}$ can be described by the criteria

$$\begin{aligned}\hat{\theta} &= f_1 + \Phi \\ \dot{\hat{\theta}} &= g_1 + \Omega\end{aligned} \tag{23}$$

where $f_1 : [0, \infty) \to \mathbb{R}^p$ is a known function such that

$$\begin{aligned}&\|f_1\| \leq \gamma_1, \\ &\|\dot{f}_1\| \leq \gamma_2 + \gamma_3\|e_1\| + \gamma_4\|e_2\| + \gamma_5\|r\|,\end{aligned} \tag{24}$$

where $\gamma_i \in \mathbb{R}$, $(i = 1,2,...,5)$ are known non-negative constants, and $\Phi\,[0,\infty) \to \mathbb{R}^p$ is a known function that satisfies the bound $\|\Phi\| \leq \rho_1(\|\bar{e}\|)\|\bar{e}\|$, where the bounding function $\rho_1 : [0,\infty) \to \mathbb{R}$ is positive, and non-decreasing, and $\bar{e} \in \mathbb{R}^{2n}$ is defined as $\bar{e} \triangleq [\ e_1^T \quad e_2^T\ ]^T$. In Eq. (23), $g_1 : [0,\infty) \to \mathbb{R}$ is a known function such that

$$\begin{aligned} &\|g_1\| \leq \delta_1, \\ &\|\dot{g}_1\| \leq \delta_2 + \delta_3\|e_1\| + \delta_4\|e_2\| + \delta_5\|r\|, \end{aligned} \tag{25}$$

where $\delta_i \in \mathbb{R}$, $(i = 1,2,...,5)$ are known non-negative constants, and $\Omega \in \mathbb{R}^p$ satisfies the bound $\|\Omega\| \leq \rho_2(\|z\|)\|z\|$ where the bounding function $\rho_2 : [0,\infty) \to \mathbb{R}$ is a positive, globally invertible, non-decreasing function, and $z \in \mathbb{R}^{3n}$ is defined as $z \triangleq [\ e_1^T \quad e_2^T \quad r^T\ ]^T$.

The adaptive update law design is flexible in the sense that asymptotic tracking is achieved using any update law that satisfies Eqs. (23) - (25). The terms $\tilde{N}_0$ and N_{B_2} introduced previously are defined as $\tilde{N}_0 \triangleq -\dot{Y}_d\Phi - Y_d\Omega$ and $N_{B_2} \triangleq -\dot{Y}_d f_1 - Y_d g_1$. The Mean Value Theorem can now be used to develop the upper bound $\left\|\tilde{N}\right\| \leq \rho(\|z\|)\|z\|$, where the bounding function $\rho : [0,\infty) \to \mathbb{R}$ is positive and non-decreasing. Based on the previous expressions, their time derivatives, and the previous inequalities, the bounds $\|N_B\| \leq \zeta_1$, $\left\|\dot{N}_{B_1}\right\| \leq \zeta_2$, and $\left\|\dot{N}_{B_2}\right\| \leq \zeta_3 + \zeta_4\|e_1\| + \zeta_5\|e_2\| + \zeta_6\|r\|$ can be developed, where $\zeta_i \in \mathbb{R}$, $(i = 1,2,...,6)$ are known positive constants.

Consider the Lyapunov function

$$V = \frac{1}{2}r^T Mr + e_1^T e_1 + \frac{1}{2}e_2^T e_2 + P.$$

Since the Lyapunov function does not contain the typical adaptive update law mismatch term $\frac{1}{2}\tilde{\theta}^T\tilde{\theta}$, the cross terms resulting from the parameter update law need to be handled separately. To handle these terms, the definition of P is modified to inject the required negative terms in the Lyapunov derivative. The auxiliary term P is defined as a Filippov solution to the differential equation

$$\dot{P} = -r^T(N_B - \beta_1 sgn(e_2)) + \beta_2\|e_1\|\|e_2\| + \beta_3\|e_2\|^2 + \beta_4\|e_2\|\|r\|, \quad P(0) = \beta_1\sum|e_{2i}(0)| - e_2(0)^T N_B(0).$$

For a complete stability analysis that yields semi-global asymptotic tracking, the reader is referred to Ref. 16.

III. RISE-based Control for Systems With Unstructured Uncertainties

III.A. RISE-based control using neural networks[17]

To illustrate how the RISE method can be used with a NN to compensate for non-LP uncertainties, reconsider the development in Eqs. (1) and (2), where the dynamics in Eq. (3) are now expressed as

$$M(q)r = F_d + S + h - u, \tag{26}$$

where $F_d, S \in \mathbb{R}^n$ are defined as

$$\begin{aligned} F_d &\triangleq M(q_d)\ddot{q}_d + V_m(q_d,\dot{q}_d)\dot{q}_d + G(q_d) + F(\dot{q}_d), \\ S &\triangleq M(q)\{V_m(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) - \ddot{q}_d + \alpha_1\dot{e}_1 + \alpha_2 e_2\} - F_d. \end{aligned}$$

Using the universal approximation property of NNs the uncertain term F_d can be represented by a multi-layer NN as

$$F_d = W^T\sigma\left(V^T x_d\right) + \varepsilon(x_d),$$

where $W \in \mathbb{R}^{(N_2+1)\times n}$ and $V \in \mathbb{R}^{(3n+1)\times N_2}$ are constant ideal weight matrices, $N_2 \in \mathbb{R}$ denotes the number of neurons in the hidden layer, $\sigma : \mathbb{R}^{N_2} \to \mathbb{R}^{N_2+1}$ is an activation function, $\varepsilon : \mathbb{R}^{3n+1} \to \mathbb{R}^n$ is the function approximation error, and $x_d \triangleq \left[\ 1 \quad q_d^T \quad \dot{q}_d^T \quad \ddot{q}_d^T\ \right]^T \in \mathbb{R}^{3n+1}$. Based on the open-loop error system in Eq. (26), the control input is designed as

$$u = -\hat{F}_d - \mu, \tag{27}$$

where $\mu \in \mathbb{R}^n$ is the RISE feedback term given in Eq. (9), and $\hat{F}_d \in \mathbb{R}^n$ denotes the NN feedforward component defined as

$$\hat{F}_d \triangleq \hat{W}^T\sigma\left(\hat{V}^T x_d\right),$$

where $\hat{W} \in \mathbb{R}^{(N_2+1)\times n}$ and $\hat{V} \in \mathbb{R}^{(3n+1)\times N_2}$ are estimates of the ideal NN weight matrices. The estimates for the ideal NN weights are generated as

$$\dot{\hat{W}} \triangleq proj\left(\Gamma_1 \hat{\sigma}' \hat{V}^T \dot{x}_d e_2^T\right), \tag{28}$$

$$\dot{\hat{V}} \triangleq proj\left(\Gamma_2 \dot{x}_d \left(\hat{\sigma}'^T \hat{W} e_2\right)^T\right), \tag{29}$$

where $\Gamma_1 \in \mathbb{R}^{(N_2+1)\times(N_2+1)}$, $\Gamma_2 \in \mathbb{R}^{(3n+1)\times(3n+1)}$ are constant positive definite gain matrices, $\hat{\sigma}' \triangleq d\sigma\left(\hat{V}^T x\right)/d\left(\hat{V}^T x\right)$, and $proj(\cdot)$ denotes a smooth projection operator.[1] The controller defined in Eq. (27) along with the adaptive update laws in Eqs. (28) and (29) yield the following closed-loop error system

$$M(q)\dot{r} = -\frac{1}{2}\dot{M}(q)r + \tilde{N} + N_d + N_B - e_2 - (k_s+1)r - \beta_1 sgn(e_2), \tag{30}$$

where $\tilde{N}, N_d, N_B \in \mathbb{R}^n$. Motivation for the segregation of terms into $\tilde{N}$, N_d and N_B is due to the different bounds on different components of the dynamics. As is typical in the RISE control development, $\tilde{N}$ contains state-dependent terms that are upper bounded via the Mean Value Theorem while N_d contains terms that can be upper bounded by a constant and whose time derivatives are also upper bounded by a constant. The components in the dynamics grouped into N_B involve the NN weight estimates and are upper bounded by a constant; however, the time derivative of these terms is dependent on e_2 and violates the traditional RISE control analysis. Typically, the candidate Lyapunov function includes an auxiliary term, P, and is shown to be positive provided that the control gains are selected according to sufficient conditions. The addition of N_B terms requires a new definition for P that results in altered sufficient gain conditions. To compensate for the N_B terms, P is defined as the Filippov solution to the following differential equation[17]

$$\dot{P} = -r^T\left(N_{B_1} + N_d - \beta_1 sgn(e_2)\right) - \dot{e}_2 N_{B_2} + \beta_2 \|e_2\|^2, \quad P(0) = \beta_1 \sum_{i=1}^{n} |e_{2i}(0)| - e_2(0)^T N(0), \tag{31}$$

where $N \triangleq N_B + N_d$, $\beta_2 \in \mathbb{R}$ is a positive constant, and N_B has been further segregated into N_{B_1}, which is rejected via the RISE feedback, and N_{B_2}, which is partially rejected by the RISE feedback and partially canceled by the adaptive update laws for the NN weight estimates. Ref. 17 establishes sufficient gain conditions for β_1 and β_2 such that $P \geq 0$, and provides a Lyapunov-based stability analysis that guarantees asymptotic tracking of the desired trajectory within a region of attraction (i.e. a semi-global result) that can be made arbitrarily large by increasing the control gains.

III.B. RISE-based control using actor-critic-based adaptation law[20]

The RISE control structure, as described in Section II.C, is able to provide asymptotic tracking in the presence of modeling uncertainties without the use of feedforward adaptive techniques. This feature can be exploited to develop adaptive controllers with improved transient performance. An actor-critic (AC) based adaptive update scheme is developed in Ref. 20 for improved tracking performance. As illustrated in Figure 1, the AC scheme is based on a RL strategy, where the performance of the actor (i.e. the feedforward part of the controller) can be improved based on a reinforcement signal from the critic. The actor and the critic are modeled as feedforward NNs, and the RISE architecture is used to obtain an asymptotic result. This section presents a brief summary of the AC-based adaptation scheme developed for a second order system in Brunovsky form.

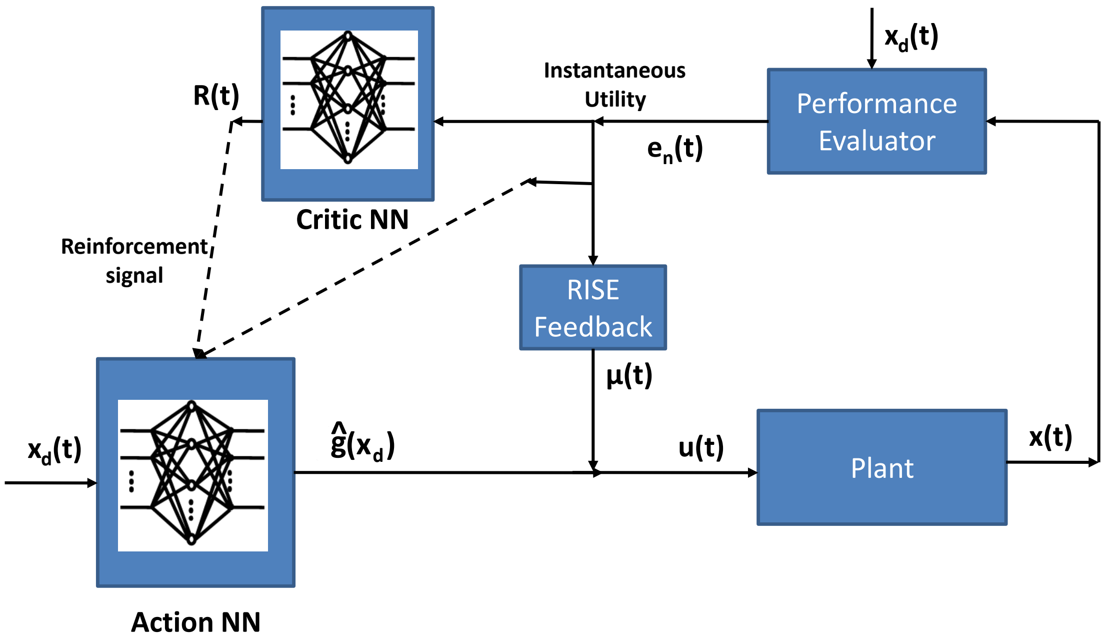

Figure 1. [20]**Architecture of the RISE-based adaptive critic controller.**

Defining $x_1 \triangleq q$, $x_2 \triangleq \dot{q}$, $g \triangleq G^{-1}f$, $U \triangleq G^{-1}u$, and $d \triangleq G^{-1}h$, the dynamical system in Eq. (12) can be expressed in second order MIMO Brunovsky form as[21]

$$\begin{aligned}\dot{x}_1 &= x_2,\\ \dot{x}_2 &= g+U+d,\\ y &= x_1,\end{aligned}$$

where the system states $x_1, x_2 \in \mathbb{R}^n$ are measurable. The control objective is to design a continuous RL-based NN controller such that the output y tracks a desired trajectory y_d. To facilitate the control design, the following assumptions are imposed on the system.[20] To facilitate the control development and stability analysis, the function $g : \mathbb{R}^{2n} \to \mathbb{R}^n$ is assumed to be second order differentiable, i.e., $g(\cdot), \dot{g}(\cdot), \ddot{g}(\cdot) \in \mathscr{L}_\infty$ if $x, \dot{x}, \ddot{x} \in \mathscr{L}_\infty$.

Using the filtered tracking error definitions from Eq. (2), the open-loop error system can be represented as

$$r = g_d + S + d + Y + U,$$

where $g_d \triangleq g(x_d)$, $x_d \triangleq [y_d^T, \dot{y}_d^T]^T \in \mathbb{R}^{2n}$, $Y \in \mathbb{R}^n$ contains known and measurable terms, and $S = g - g_d \in \mathbb{R}^n$. The unknown term g_d is represented using a multi-layer neural network as $g_d = W_a^T \sigma_a(V_a^T x_a) + \varepsilon(x_a)$, where $x_a \triangleq \left[1, x_d^T\right]^T \in \mathbb{R}^{2n+1}$ is the input to the NN, $W_a \in \mathbb{R}^{(N_a+1)\times n}$ and $V_a \in \mathbb{R}^{(2n+1)\times N_a}$ are the constant ideal weights, $N_a \in \mathbb{R}$ denotes the number of neurons in the hidden layer, $\sigma_a : \mathbb{R}^{N_a} \to \mathbb{R}^{(N_a+1)}$ is the activation function, and $\varepsilon : \mathbb{R}^{2n+1} \to \mathbb{R}^n$ is the function approximation error. The NN is referred to as the action NN or the associative search element (ASE),[22] that generates an estimate of the system dynamics which can be used to generate a feedforward control signal. Using an approximation $\hat{g} \triangleq \hat{W}_a^T \sigma(\hat{V}_a^T x_a)$, the control input is designed as $U \triangleq -Y - \hat{g} - \mu$, where $\mu \in \mathbb{R}^m$ is the RISE feedback term defined in Eq. (9). The AC-based update laws for the action NN are given by

$$\begin{aligned}\dot{\hat{W}}_a &\triangleq \Gamma_{aw} proj(\alpha_2 \sigma_a'(\hat{V}_a^T x_a)\hat{V}_a^T \dot{x}_a e_2^T\\ &\quad + \sigma_a(\hat{V}_a^T x_a) R \hat{W}_c^T \sigma_c'(\hat{V}_c^T e_2)\hat{V}_c^T),\\ \dot{\hat{V}}_a &= \Gamma_{av} proj(\alpha_2 \dot{x}_a e_2^T \hat{W}_a^T \sigma_a'(\hat{V}_a^T x_a)\\ &\quad + x_a R \hat{W}_c^T \sigma_c'(\hat{V}_c^T e_2)\hat{V}_c^T \hat{W}_a^T \sigma_a'(\hat{V}_a^T x_a)),\end{aligned}$$

where $\hat{W}_a \in \mathbb{R}^{(N_a+1)\times n}$ and $\hat{V}_a \in \mathbb{R}^{(2n+1)\times N_a}$ are estimates of the ideal weights for the actor NN, $\hat{W}_c \in \mathbb{R}^{(N_c+1)}$ and $\hat{V}_c \in \mathbb{R}^{n\times N_c}$ are estimates of the ideal weights, $\sigma_c : \mathbb{R}^{N_c} \to \mathbb{R}^{(N_c+1)}$ is the activation function, and $N_c \in \mathbb{R}$ is the number of hidden layer neurons for the critic NN, $\Gamma_{aw} \in \mathbb{R}^{(N_a+1)\times(N_a+1)}$ and $\Gamma_{av} \in \mathbb{R}^{(2n+1)\times(2n+1)}$ are constant positive definite gain matrices and $R \in \mathbb{R}$ is the reinforcement signal generated by the critic, given by $R \triangleq \hat{W}_c^T \sigma_c(\hat{V}_c^T e_2) + \psi$, where ψ is a Filippov solution to the differential equation $\dot{\psi} = \hat{W}_c^T \sigma_c'(\hat{V}_c^T e_2)\hat{V}_c^T(\mu_a + \alpha_2 e_2) - k_c R - \beta_2 sgn(R)$, where $k_c, \beta_2 \in \mathbb{R}$ are constant positive control gains. The critic design uses the filtered tracking error e_2 as an instantaneous utility function[20] and attempts to minimize the instantaneous error. The resulting update laws are given by

$$\dot{\hat{W}}_c = -\Gamma_{cw} proj(\sigma_c(\hat{V}_c^T e_n)R + \hat{W}_c),$$
$$\dot{\hat{V}}_c = -\Gamma_{cv} proj(e_n \hat{W}_c^T \sigma_c'(\hat{V}_c^T e_n)R + \hat{V}_c),$$

where $\Gamma_{cw}, \Gamma_{cv} \in \mathbb{R}$ are constant positive gains.

Semi-global asymptotic tracking of the desired trajectory is established in Ref. 20 using Lyapunov-based techniques similar to those detailed in the previous sections. The performance benefit over the NN-based controller detailed in Section III.A is verified experimentally by performing repeated trials on a two-link robot manipulator. Figures 2 and 3 illustrate the error and control trajectories for one of the trials. The results indicate that the mean RMS the position tracking errors for Link 1 and Link 2 are approximately 14% and 7% smaller for the proposed actor-critic-based RISE controller. For a complete description of the test bed, and a statistical analysis, see Ref. 20.

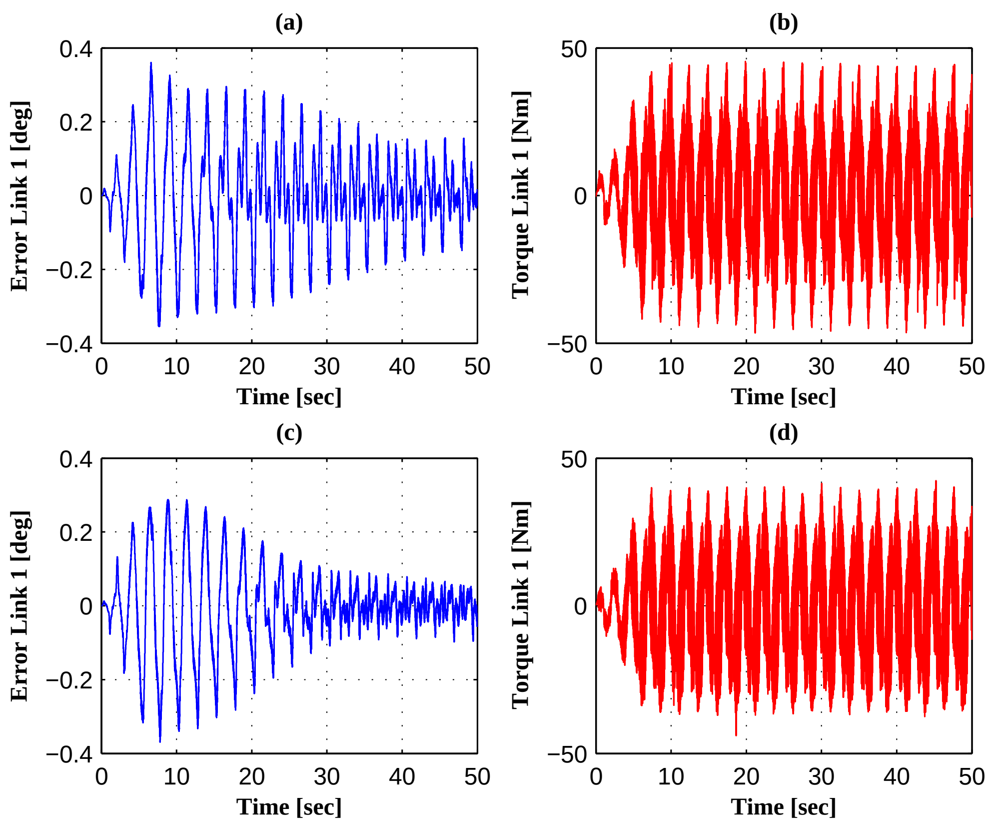

Figure 2. [20]**Comparison of tracking errors and torques between NN+RISE and AC+RISE for link 1 (a) Tracking error with NN+RISE, (b) Control Torque with NN+RISE, (c) Tracking error with AC+RISE, (d) Control Torque with AC+RISE**

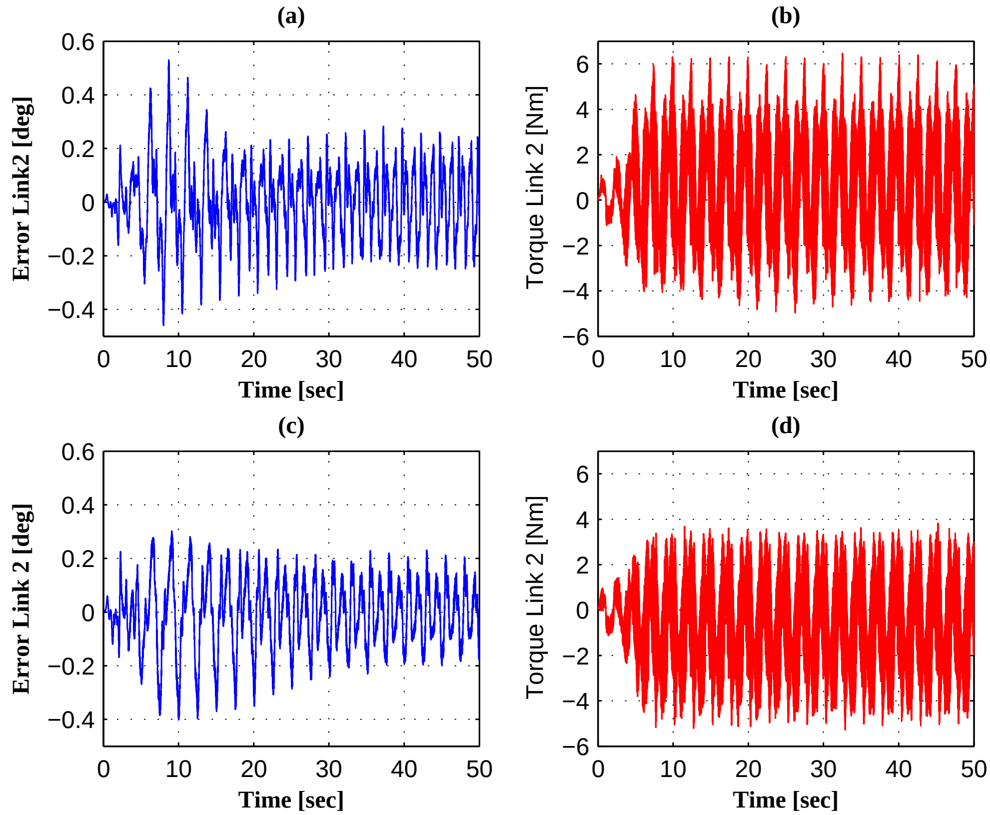

Figure 3. [20]Comparison of tracking errors and torques between NN+RISE and AC+RISE for link 2 (a) Tracking error with NN+RISE, (b) Control Torque with NN+RISE, (c) Tracking error with AC+RISE, (d) Control Torque with AC+RISE

The controller described in this section uses the RISE architecture in conjunction with actor-critic-based adaptive update laws to improve the performance of non-LP adaptive systems. The design flexibility that the RISE architecture offers is utilized in this result to combine ideas from adaptive control literature and RL literature to develop innovative RL-based learning schemes. RL-based methods have been shown to be effective methods for on-line synthesis of near-optimal controllers. These techniques attempt to generate an approximate solution to the Hamilton-Jacobi-Bellman equation, by minimizing the Bellman error.[23]

IV. Conclusion

This paper presents a brief overview of the RISE control strategy and its application in adaptive control problems. The RISE control structure is designed to yield asymptotic results and, when combined with feedforward adaptive techniques, can exhibit improved transient performance.

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