

Unknown Time-Varying Input Delay Compensation for Uncertain Nonlinear Systems

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Abstract

A tracking controller is developed for a class of uncertain nonlinear systems subject to unknown time-varying input delay and additive disturbances. A novel filtered error signal is designed using the past states in a finite integral over a constant estimated delay interval. The maximum tolerable error between unknown time-varying delay and a constant estimate of the delay is determined to establish uniformly ultimately bounded convergence of the tracking error to the origin. The controller development is based on an approach which uses Lyapunov-Krasovskii functionals to analyze the effects of unknown sufficiently slowly time-varying input delays. A stability analysis is provided to prove ultimate boundedness of the tracking error signals. Numerical simulation results illustrate the performance of the developed robust controller.

Key words: Input Delay, Robust Control, Nonlinear Control

1 Introduction

Time delay commonly exists in many engineering applications such as master-slave robots, haptic systems, chemical systems and biological systems. The system dynamics, communication over a network, and sensing with associated sensor processing (e.g., image-based feedback) can induce time delays that can result in decreased performance and loss of stability. Time delays in physical systems are often time-varying. For example, the input delay in neuromuscular electrical stimulation applications often changes with muscle fatigue [1, 2], communication delays in wireless networks change with the distance between the communicating agents, etc. Motivated by such practical engineering challenges, numerous research efforts have focused on designing controllers to compensate time delay disturbances effects.

Research in recent years has focused on developing controllers that provide stability for systems with delays in the closed-loop dynamics. Smith's pioneering work [3], Arstein's model reduction [4], and the finite spectrum approach [5] have heavily influenced the methods of designing controllers that compensate the effects of delays.

In recent years, research has focused on systems that ex-

perience a known delay in the control input. The works in [6–8] develop robust controllers which compensate for known input time delay for systems with linear plant dynamics. Compensation of input delay disturbances for nonlinear plant dynamics is addressed in prominent works such as [9–17] for nonlinear plant dynamics affected by external disturbances and in [18–20] for plant dynamics without external disturbances. However, the controllers in [9–20] require exact knowledge of the time delay duration. In practice, the duration of an input time delay can be challenging to determine for some applications, therefore, it is necessary to develop new controllers that do not require exact knowledge of the time delay.

Since uncertainty in the delay can lead to unpredictable closed-loop performance (potentially even instabilities), several recent results have been developed which do not assume that the delay is exactly known. Compensation for unknown input delay is investigated in [21–31] for systems with exactly known dynamics and [32–36] for systems with uncertain dynamics. However, all of the controllers in [21–36] are developed for linear plant dynamics. The works in [37–41] develop controllers for plants with nonlinear dynamics and an unknown input delay, but require exact model knowledge of the nonlinear dynamics. The controller designed in [42] compensates for Takagi-Sugeno fuzzy systems and unknown actuation delay duration by using a memoryless observer and a fuzzy parallel distributed integral compensator for nonlinear, uncertain dynamics. However, the controller

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in [42] is designed for output regulation and does not address the output tracking problem. There remains a need for a tracking controller that can compensate for the effects of unknown time-varying input delays for a class of uncertain nonlinear systems.

When uncertain nonlinear dynamics are present, the control design is significantly more challenging than when linear or exactly known nonlinear dynamics are present. For example, in general, if the system states evolve according to linear dynamics, the linear behavior can be exploited to predict the system response over the delay interval. Exact knowledge of the dynamics facilitates the ability to predict the state transition for nonlinear systems. For uncertain nonlinear systems, the state transition is significantly more difficult to predict, especially if the delay interval is also unknown and/or time-varying. Given the difficulty in predicting the state transition, the contribution in this paper (and in [11, 43]) is to treat the input delay and dynamic uncertainty as a disturbance and develop a robust controller that can compensate for these effects.

Recently, Fischer et al. presented a robust controller for uncertain nonlinear systems with additive disturbances subject to slowly varying input delay in [11], where it is assumed that the input delay duration is measurable and the absolute value of the second derivative of the delay is bounded by a known constant. The approach in this study extends our previous work in [11] by using a novel filtered error signal to compensate for an unknown slowly varying input delay for uncertain nonlinear systems affected by additive disturbances. In [11], a filtered error signal defined as the finite integral of the actuator signals over the delay interval is used to obtain a delay-free expression for the control input in the closed-loop error system. However, the computation of the finite integral requires exact knowledge of the input delay. In this study, a novel filtered error signal is designed using the past states in a finite integral over a constant estimated delay interval to cope with the lack of delay knowledge, which requires a significantly different stability analysis that takes advantage of Lyapunov-Krasovskii functionals. Techniques used in this study provide relaxed requirements of the delay measurement and obviate the need for a bound of the absolute value of the second derivative of the delay. It is assumed that the estimated input delay is selected sufficiently close to the actual time-varying input delay. That is, there are robustness limitations, which can be relaxed with more knowledge about the time-delay. Because it is feasible to obtain lower and upper bounds for the input delay in many applications [44], it is feasible to select a delay estimate in an appropriate range. New sufficient conditions for stability are based on the length of the estimated delay as well as the maximum tolerable error between the actual and estimated input delay. A Lyapunov-based stability analysis is used to prove ultimate boundedness of the error signals. Numerical simulation results demonstrate

the performance of the robust controller.

2 Dynamic System

Consider a class of n^{th} -order nonlinear systems

$$\begin{aligned} \dot{x}_i &= x_{i+1}, \quad i = 1, \dots, n-1, \\ \dot{x}_n &= f(X, t) + d + u(t - \tau), \end{aligned} \quad (1)$$

where $x_i \in \mathbb{R}^m$, $i = 1, \dots, n$ are the measurable system states, $X = [x_1^T, x_2^T, \dots, x_n^T]^T \in \mathbb{R}^{mn}$, $u \in \mathbb{R}^m$ is the control input, $f: \mathbb{R}^{mn} \times [t_0, \infty) \rightarrow \mathbb{R}^m$ is an uncertain nonlinear function, $d: [t_0, \infty) \rightarrow \mathbb{R}^m$ denotes sufficiently smooth unknown additive disturbance (e.g., unmodeled effects), and $\tau: [t_0, \infty) \rightarrow \mathbb{R}$ denotes a time-varying unknown positive time delay¹, where t_0 is the initial time. Throughout the paper, delayed functions are denoted as

$$h_\tau \triangleq \begin{cases} h(t - \tau) & t - \tau \geq t_0 \\ 0 & t - \tau < t_0. \end{cases}$$

The dynamic model of the system in (1) can be rewritten as

$$\dot{x}_1^{(n)} = f(X, t) + d + u(t - \tau), \quad (2)$$

where the superscript (n) denotes the n^{th} time derivative. In addition, the dynamic model of the system in (1) satisfies the following assumptions.

Assumption 1 *The function f and its first and second partial derivatives are bounded on each subset of their domain of the form $\Xi \times [t_0, \infty)$, where $\Xi \subset \mathbb{R}^{mn}$ is compact and for any given Ξ , the corresponding bounds are known².*

Assumption 2 [45] *The nonlinear additive disturbance term and its first time derivative (i.e., d, \dot{d}) exist and are bounded by known positive constants.*

Assumption 3 *The reference trajectory $x_r \in \mathbb{R}^m$ is designed such that the derivatives $x_r^{(i)}, \forall i = 0, 1, \dots, (n+2)$ exist and are bounded by known positive constants.*

¹ The developed method can be extended to the case of multiple delays. Assumption 4 can be modified for the case of multiple delays by redefining the delayed input vector and using the maximum input delay instead of the actual delay bound such that $\max\{\tau_1, \tau_2, \dots, \tau_m\} < \Upsilon$. To obviate the requirement of exact knowledge of the time delay dynamics in the stability analysis and introducing new Lyapunov-Krasovskii functionals for each input delay, the closed-loop dynamics can be revised in terms of $\dot{u}_{\hat{\tau}}, (\dot{u}_\tau - \dot{u}_{\hat{\tau}})$ instead of the terms $\dot{u}_{\hat{\tau}}, \dot{u}_\tau, (\dot{u}_\tau - \dot{u}_{\hat{\tau}})$. In this paper, single time-varying input delay is considered for ease of exposition.

² Given a compact set $\Xi \subset \mathbb{R}^{mn}$, the bounds of $f, \frac{\partial f(X,t)}{\partial X}, \frac{\partial f(X,t)}{\partial t}, \frac{\partial^2 f(X,t)}{\partial^2 X}, \frac{\partial^2 f(X,t)}{\partial X \partial t}$, and $\frac{\partial^2 f(X,t)}{\partial t^2}$ over Ξ are assumed to be known.

Assumption 4 *The input delay is bounded such that $\tau(t) < \Upsilon$ for all $t \in \mathbb{R}$, differentiable, and slowly varying such that $|\dot{\tau}| < \varphi < 1$ for all $t \in \mathbb{R}$, where $\varphi, \Upsilon \in \mathbb{R}$ are known positive constants. Additionally, a sufficiently accurate constant estimate $\hat{\tau} \in \mathbb{R}$ of τ is available such that $|\tilde{\tau}| \leq \bar{\tau}$, for all $t \in \mathbb{R}$, where $\tilde{\tau} \triangleq \tau - \hat{\tau}$, and $\bar{\tau} \in \mathbb{R}$ is a known positive constant³. Furthermore, it is assumed that the system in (1) does not escape to infinity during the time interval $[t_0, t_0 + \Upsilon]$.*

3 Control Development

The objective of the control design is to develop a continuous controller which ensures that the state x_1 of the delayed system in (2) tracks a reference trajectory, x_r .

To quantify the control objective, a tracking error, denoted by $e_1 \in \mathbb{R}^m$, is defined as

$$e_1 \triangleq x_r - x_1. \quad (3)$$

To facilitate the subsequent analysis, auxiliary tracking error signals, denoted by $e_i \in \mathbb{R}^m$, $i = 2, 3, \dots, n$, are defined as [46]

$$e_2 \triangleq \dot{e}_1 + e_1, \quad (4)$$

$$e_3 \triangleq \dot{e}_2 + e_2 + e_1, \quad (5)$$

\vdots

$$e_n \triangleq \dot{e}_{n-1} + e_{n-1} + e_{n-2}. \quad (6)$$

A general expression of e_i for $i = 2, 3, \dots, n$ can be written as

$$e_i = \sum_{j=0}^{i-1} a_{i,j} e_1^{(j)}, \quad (7)$$

where $a_{i,j} \in \mathbb{R}$ are defined as⁴ [47]

$$a_{i,0} \triangleq \frac{1}{\sqrt{5}} \left(\left(\frac{1+\sqrt{5}}{2} \right)^i - \left(\frac{1-\sqrt{5}}{2} \right)^i \right), \quad \forall i = 2, 3, \dots, n,$$

$$a_{i,j} \triangleq \sum_{p=1}^{i-1} a_{i-p-j+1,0} a_{p+j-1,j-1}, \quad \forall i=3,4,\dots,n, \forall j=1,2,\dots,(i-2).$$

To obtain a delay-free control expression for the input in the closed-loop error system, an auxiliary tracking error

³ Since the maximum tolerable error, $\bar{\tau}$, and the estimate of actual delay, $\hat{\tau}$, are known, the maximum tolerable input delay can be determined. Because the bounds on the input delay are feasible to obtain in many applications [44], Assumption 4 is reasonable.

⁴ It should be noted that $a_{i,i-1} = 1$, $\forall i = 1, 2, \dots, n$.

signal, denoted by $e_u \in \mathbb{R}^m$, is defined as

$$e_u \triangleq - \int_{t-\hat{\tau}}^t \dot{u}(\theta) d\theta. \quad (8)$$

It should be emphasized that the estimate of τ , denoted by $\hat{\tau}$, is required instead of exact knowledge of τ in the control design. For example, the constant estimate $\hat{\tau}$ may be selected to best approximate the mean of τ . Based on the subsequent stability analysis, the following continuous robust controller is designed as

$$u \triangleq k(e_n - e_n(t_0)) + v, \quad (9)$$

where $e_n(t_0) \in \mathbb{R}^m$ is the initial error signal and $v \in \mathbb{R}^m$ is the solution to the differential equation

$$\dot{v} = k(\Lambda e_n + \alpha e_u), \quad (10)$$

where $k, \Lambda, \alpha \in \mathbb{R}^{m \times m}$ are constant, diagonal, positive definite gain matrices.

4 Stability Analysis

To facilitate the stability analysis an auxiliary tracking error signal, denoted by $r \in \mathbb{R}^m$, is defined as⁵

$$r \triangleq \dot{e}_n + \Lambda e_n + \alpha e_u, \quad (11)$$

The open loop dynamics for r can be obtained by substituting the first time derivatives of (2) and (8), the second time derivative of (7) with $i = n$, and the $(n+1)^{\text{th}}$ time derivative of (3) into (11) as

$$\begin{aligned} \dot{r} = & -\dot{f}(X, \dot{X}, t) - \dot{d} + \sum_{j=0}^{n-2} a_{n,j} e_1^{(j+2)} + x_r^{(n+1)} \\ & - \alpha \dot{u} + \alpha \dot{u}_{\hat{\tau}} - (1 - \dot{\tau}) \dot{u}_{\tau} + \Lambda \dot{e}_n. \end{aligned} \quad (12)$$

Substituting the first time derivative of the controller in (9) into (12), the closed-loop error system for r can be obtained as

$$\begin{aligned} \dot{r} = & -\dot{f}(X, \dot{X}, t) - \dot{d} + \sum_{j=0}^{n-2} a_{n,j} e_1^{(j+2)} + x_r^{(n+1)} + \Lambda \dot{e}_n \\ & - \alpha k r + (\alpha - I + \dot{\tau} I) k r_{\tau} + \alpha (\dot{u}_{\hat{\tau}} - \dot{u}_{\tau}), \end{aligned} \quad (13)$$

where $I \in \mathbb{R}^{m \times m}$ is the identity matrix. The stability analysis can be facilitated by segregating the terms in (13) that can be upper bounded by a state-dependent

⁵ Since \dot{e}_n is not measurable, r cannot be used in the control design.

function and terms that can be upper bounded by a constant, such that

$$\begin{aligned} \dot{r} &= -\alpha kr + (\alpha - I + \hat{\tau}I) kr_\tau + \alpha (\dot{u}_\tau - \dot{u}_r) \\ &+ \tilde{N} + N_r - e_n. \end{aligned} \quad (14)$$

The auxiliary functions $\tilde{N} \in \mathbb{R}^m$ and $N_r \in \mathbb{R}^m$ are defined as

$$\begin{aligned} \tilde{N} &\triangleq -\dot{f}(X, \dot{X}, t) + \dot{f}(X_r, \dot{X}_r, t) + \sum_{j=0}^{n-2} a_{n,j} e_1^{(j+2)} \\ &+ \Lambda \dot{e}_n + e_n, \end{aligned} \quad (15)$$

$$N_r \triangleq -\dot{f}(X_r, \dot{X}_r, t) - \dot{d} + x_r^{(n+1)}, \quad (16)$$

where $X_r \triangleq \left[x_r^T, \dot{x}_r^T, \dots, (x_r^{(n-1)})^T \right]^T \in \mathbb{R}^{mn}$.

Remark 1 Based on Assumptions 2 and 3, N_r is upper bounded as

$$\sup_{t \in \mathbb{R}} \|N_r\| \leq \zeta_{N_r}, \quad (17)$$

where $\zeta_{N_r} \in \mathbb{R}$ is a known positive constant.

Remark 2 An upper bound can be obtained for (15) using Assumption 1 and the Lemma 5 in [48] as

$$\|\tilde{N}\| \leq \rho(\|z\|) \|z\|, \quad (18)$$

where ρ is a positive, radially unbounded⁶, and strictly increasing function, and $z \in \mathbb{R}^{(n+2)m}$ is a vector of error signals defined as

$$z \triangleq [e_1^T, e_2^T, \dots, e_n^T, e_u^T, r^T]^T. \quad (19)$$

To facilitate the subsequent stability analysis, auxiliary bounding constants $\sigma, \delta \in \mathbb{R}$ are defined as

$$\begin{aligned} \sigma &\triangleq \min \left\{ 1, \left(1 - \frac{\epsilon_2}{2}\right), \left(\underline{\Lambda} - \left(\frac{\bar{\alpha}}{2\epsilon_1} + \frac{1}{2\epsilon_2}\right)\right), \right. \\ &\left. \frac{k\alpha}{8}, \left(\frac{\omega_2}{4\hat{\tau}} - \bar{\alpha}\epsilon_1\right) \right\}, \end{aligned} \quad (20)$$

$$\begin{aligned} \delta &\triangleq \frac{1}{2} \min \left\{ \frac{\sigma}{2}, \frac{\omega_2 k^3 \alpha (1-\varphi)}{4(\bar{k}(\bar{\alpha} + \varphi - 1))^2}, \right. \\ &\left. \frac{\omega_2 k^2 \bar{\alpha} \epsilon_1}{4\omega_1^2 k^2}, \frac{1}{4(\bar{\tau} + \hat{\tau})} \right\}, \end{aligned} \quad (21)$$

⁶ For some classes of systems, the bounding function ρ can be selected as a constant. For those systems, a global uniformly ultimately bounded result can be obtained as described in Remark 3.

where $\underline{\Lambda}, \underline{k}, \underline{\alpha} \in \mathbb{R}$ denote the minimum eigenvalues of Λ, k, α , respectively, $\bar{k}, \bar{\alpha} \in \mathbb{R}$ denote the maximum eigenvalues of k and α , respectively, and $\omega_i, \epsilon_i \in \mathbb{R}, i = 1, 2$, are known, selectable, positive constants.

Let the functions $Q_1, Q_2, Q_3 \in \mathbb{R}$ be defined as

$$Q_1 \triangleq \frac{(\omega_1 \bar{k})^2}{\bar{\alpha} \epsilon_1} \int_{t-\hat{\tau}}^t \|r(\theta)\|^2 d\theta, \quad (22)$$

$$Q_2 \triangleq \frac{(\bar{k}(\bar{\alpha} + \varphi - 1))^2}{k\alpha(1-\varphi)} \int_{t-\tau}^t \|r(\theta)\|^2 d\theta, \quad (23)$$

$$Q_3 \triangleq \omega_2 \int_{t-(\bar{\tau}+\hat{\tau})}^t \int_s^t \|\dot{u}(\theta)\|^2 d\theta ds, \quad (24)$$

and let $y \in \mathbb{R}^{(n+2)m+3}$ be defined as

$$y \triangleq \left[z, \sqrt{Q_1}, \sqrt{Q_2}, \sqrt{Q_3} \right]^T. \quad (25)$$

For use in the following stability analysis, let

$$\mathcal{D}_1 \triangleq \left\{ y \in \mathbb{R}^{(n+2)m+3} \mid \|y\| < \chi_1 \right\}, \quad (26)$$

where $\chi_1 \triangleq \inf \left\{ \rho^{-1} \left(\left[\sqrt{\frac{\sigma k \alpha}{2}}, \infty \right) \right) \right\}$. Provided $\|z(\eta)\| < \gamma, \forall \eta \in [t_0, t]$, (14) and the fact that $\dot{u} = kr$ can be used to conclude that $\ddot{u} < M$, where γ and M ⁷ are positive constants. Let $\mathcal{D} \triangleq \mathcal{D}_1 \cap (B_\gamma \cap \mathbb{R}^{(n+2)m+3})$ where B_γ denotes a closed ball of radius γ centered at the origin and let

$$\mathcal{S}_{\mathcal{D}} \triangleq \{y \in \mathcal{D} \mid \|y\| < \chi_2\} \quad (27)$$

denote the domain of attraction, where⁸ $\chi_2 \triangleq \sqrt{\frac{\min\{\frac{1}{2}, \frac{\omega_1}{2}\}}{\max\{1, \frac{\omega_1}{2}\}}} \inf \left\{ \rho^{-1} \left(\left[\sqrt{\frac{\sigma k \alpha}{2}}, \infty \right) \right) \right\}$.

Theorem 1 Given the dynamics in (1), the controller given in (9) and (10) ensures uniformly ultimately bounded tracking in the sense that

$$\limsup_{t \rightarrow \infty} \|e_1(t)\| \leq \left(\sqrt{\frac{\max\{1, \frac{\omega_1}{2}\}}{\min\{\frac{1}{2}, \frac{\omega_1}{2}\}}} \right)$$

⁷ The subsequent analysis does not assume that the inequality $\ddot{u} < M$ holds for all time. The subsequent analysis only exploits the fact that provided $\|z(\eta)\| < \gamma, \forall \eta \in [t_0, t]$, then $\ddot{u} < M$.

⁸ For a set A , the inverse image $\rho^{-1}(A)$ is defined as $\rho^{-1}(A) \triangleq \{a \mid \rho(a) \in A\}$

$$\sqrt{\frac{(2\zeta_{N_r}^2 + \underline{\alpha} \underline{k} \bar{\alpha} \bar{\tau}^2 M^2)}{2\underline{\alpha} \underline{k} \delta}}, \quad (28)$$

provided that $y(t_0) \in \mathcal{S}_{\mathcal{D}}$ and that the control gains are selected sufficiently large based on the initial conditions of the system such that the following sufficient conditions are satisfied^{9 10}

$$\begin{aligned} \omega_2 &> 4\bar{\alpha}\epsilon_1\hat{\tau}, \\ \underline{\Lambda} &> \frac{\bar{\alpha}}{2\epsilon_1} + \frac{1}{2\epsilon_2}, \quad 2 > \epsilon_2, \\ &\left(\frac{\frac{\underline{\alpha} \underline{k}}{8} - \frac{2(\omega_1 \bar{k})^2}{\bar{\alpha}\epsilon_1} - \frac{(\bar{k}(\bar{\alpha} + \varphi - 1))^2}{\underline{\alpha} \underline{k}(1 - \varphi)}}{\omega_2 \bar{k}^2} \right. \\ &\quad \left. - \frac{\omega_2 \hat{\tau} \bar{k}^2 + \frac{\bar{\alpha}}{2}}{\omega_2 \bar{k}^2} \right) \geq \bar{\tau}, \\ \chi_2 &> \left(\frac{2\zeta_{N_r}^2 + \underline{\alpha} \underline{k} \bar{\alpha} \bar{\tau}^2 M^2}{2\underline{\alpha} \underline{k} \delta} \right)^{\frac{1}{2}}. \end{aligned} \quad (29)$$

PROOF. Let $V : \mathcal{D} \rightarrow \mathbb{R}$ be a continuously differentiable Lyapunov function candidate defined as

$$V \triangleq \frac{1}{2} \sum_{i=1}^n e_i^T e_i + \frac{1}{2} r^T r + \frac{\omega_1}{2} e_u^T e_u + \sum_{i=1}^3 Q_i. \quad (30)$$

In addition, the following upper bound can be provided for Q_3

$$\begin{aligned} Q_3 &\leq \omega_2 (\bar{\tau} + \hat{\tau}) \sup_{s \in [t - (\bar{\tau} + \hat{\tau}), t]} \left[\int_s^t \|\dot{u}(\theta)\|^2 ds, \right] \\ &\leq \omega_2 (\bar{\tau} + \hat{\tau}) \int_{t - (\bar{\tau} + \hat{\tau})}^t \|\dot{u}(\theta)\|^2 d\theta. \end{aligned} \quad (31)$$

By applying Leibniz Rule, the time derivatives of (22)-(24) can be obtained as

$$\dot{Q}_1 = \frac{(\omega_1 \bar{k})^2}{\bar{\alpha}\epsilon_1} \left(\|r\|^2 - \|r_{\hat{\tau}}\|^2 \right), \quad (32)$$

$$\dot{Q}_2 = \frac{(\bar{k}(\bar{\alpha} + \varphi - 1))^2}{\underline{k} \underline{\alpha} (1 - \varphi)} \left(\|r\|^2 - (1 - \dot{\tau}) \|r_{\tau}\|^2 \right), \quad (33)$$

⁹ To achieve a small tracking error for the case of a large value of ζ_{N_r} (i.e., fast dynamics with large disturbances), large gains, small delay, and a better estimate of the delay are required.

¹⁰ By choosing α close to $1 - \varphi$, sufficiently small ω_1 and ϵ_1 , and a sufficiently large Λ , the gain conditions can be expressed in terms of k , $\hat{\tau}$, $\bar{\tau}$, φ . The gain k can then be selected provided $\hat{\tau}$, $\bar{\tau}$, φ are small enough.

$$\dot{Q}_3 = \omega_2 \left((\bar{\tau} + \hat{\tau}) \bar{k} \|r\|^2 - \int_{t - (\bar{\tau} + \hat{\tau})}^t \|\dot{u}(\theta)\|^2 d\theta \right). \quad (34)$$

Based on (30), the following inequalities can be developed:

$$\min \left\{ \frac{1}{2}, \frac{\omega_1}{2} \right\} \|y\|^2 \leq V(y) \leq \max \left\{ 1, \frac{\omega_1}{2} \right\} \|y\|^2. \quad (35)$$

The time derivative of the first term in (30) can be obtained by using (4)-(6), (11), and the definition of e_i in (7) for $i = n$, as

$$\sum_{i=1}^n e_i^T \dot{e}_i = - \sum_{i=1}^{n-1} e_i^T e_i - e_n^T \Lambda e_n + e_{n-1}^T e_n - e_n^T \alpha e_u + e_n^T r. \quad (36)$$

By using (8), (14), (32)-(34), and (36), the time derivative of (30) can be determined as

$$\begin{aligned} \dot{V} &= - \sum_{i=1}^{n-1} e_i^T e_i - e_n^T \Lambda e_n + e_{n-1}^T e_n - e_n^T \alpha e_u + e_n^T r \\ &\quad + r^T (-\alpha k r + (\alpha - I + \dot{\tau} I) k r_{\tau} + \alpha (\dot{u}_{\hat{\tau}} - \dot{u}_{\tau})) \\ &\quad + r^T (\tilde{N} + N_r - e_n) \\ &\quad + \omega_1 e_u^T (k r_{\hat{\tau}} - k r) \\ &\quad + \frac{(\omega_1 \bar{k})^2}{\bar{\alpha}\epsilon_1} \left(\|r\|^2 - \|r_{\hat{\tau}}\|^2 \right) \\ &\quad + \frac{(\bar{k}(\bar{\alpha} + \varphi - 1))^2}{\underline{k} \underline{\alpha} (1 - \varphi)} \left(\|r\|^2 - (1 - \dot{\tau}) \|r_{\tau}\|^2 \right) \\ &\quad + \omega_2 \left((\bar{\tau} + \hat{\tau}) \bar{k}^2 \|r\|^2 - \int_{t - (\bar{\tau} + \hat{\tau})}^t \|\dot{u}(\theta)\|^2 d\theta \right). \end{aligned} \quad (37)$$

After canceling common terms and using Assumption 4, the expression in (37) can be upper bounded as

$$\begin{aligned} \dot{V} &\leq - \sum_{i=1}^{n-1} \|e_i\|^2 - \underline{\Lambda} \|e_n\|^2 + |e_{n-1}^T e_n| + \bar{\alpha} |e_n^T e_u| \\ &\quad + \bar{\alpha} |r^T (\dot{u}_{\hat{\tau}} - \dot{u}_{\tau})| - \underline{\alpha} \underline{k} \|r\|^2 \\ &\quad + r^T \tilde{N} + \|r\| \zeta_{N_r} + \bar{k} |\bar{\alpha} + \varphi - 1| |r^T r_{\tau}| \\ &\quad + \omega_1 \bar{k} (\|e_u\| \|r_{\hat{\tau}}\| + \|e_u\| \|r\|) \\ &\quad + \frac{(\omega_1 \bar{k})^2}{\bar{\alpha}\epsilon_1} \left(\|r\|^2 - \|r_{\hat{\tau}}\|^2 \right) \\ &\quad + \frac{(\bar{k}(\bar{\alpha} + \varphi - 1))^2}{\underline{k} \underline{\alpha} (1 - \varphi)} \left(\|r\|^2 - (1 - \dot{\tau}) \|r_{\tau}\|^2 \right) \end{aligned}$$

$$+ \omega_2 \left((\bar{\tau} + \hat{\tau}) \bar{k}^2 \|r\|^2 - \int_{t-(\bar{\tau}+\hat{\tau})}^t \|\dot{u}(\theta)\|^2 d\theta \right). \quad (38)$$

Using Young's Inequality the following inequalities can be obtained

$$|e_n^T e_u| \leq \frac{1}{2\epsilon_1} \|e_n\|^2 + \frac{\epsilon_1}{2} \|e_u\|^2, \quad (39)$$

$$|e_{n-1}^T e_n| \leq \frac{\epsilon_2}{2} \|e_{n-1}\|^2 + \frac{1}{2\epsilon_2} \|e_n\|^2, \quad (40)$$

$$|r^T (\dot{u}_{\hat{\tau}} - \dot{u}_\tau)| \leq \frac{1}{2} \|r\|^2 + \frac{1}{2} \|\dot{u}_{\hat{\tau}} - \dot{u}_\tau\|^2. \quad (41)$$

After completing the squares for the cross terms containing r and $r_{\hat{\tau}}$, substituting the time derivative of (9) and (18), (39)-(41) into (38), and using Assumption 4, the following upper bound can be obtained

$$\begin{aligned} \dot{V} \leq & - \sum_{i=1}^{n-2} \|e_i\|^2 - \left(1 - \frac{\epsilon_2}{2}\right) \|e_{n-1}\|^2 \\ & - \left(\underline{\Lambda} - \left(\frac{\bar{\alpha}}{2\epsilon_1} + \frac{1}{2\epsilon_2}\right)\right) \|e_n\|^2 \\ & + \bar{\alpha}\epsilon_1 \|e_u\|^2 - \frac{\alpha k}{8} \|r\|^2 \\ & - \left(\frac{\alpha k}{8} - \kappa\right) \|r\|^2 + \frac{1}{\alpha k} \rho^2 (\|z\|) \|z\|^2 \\ & + \frac{1}{\alpha k} \zeta_{N_r}^2 + \frac{\bar{\alpha} \|\dot{u}_{\hat{\tau}} - \dot{u}_\tau\|^2}{2} \\ & - \omega_2 \int_{t-(\bar{\tau}+\hat{\tau})}^t \|\dot{u}(\theta)\|^2 d\theta, \end{aligned} \quad (42)$$

where $\kappa \triangleq \frac{2(\omega_1 \bar{k})^2}{\bar{\alpha}\epsilon_1} + \frac{(\bar{k}(\bar{\alpha} + \varphi - 1))^2}{\alpha k(1-\varphi)} + \omega_2 (\bar{\tau} + \hat{\tau}) \bar{k}^2 + \frac{\bar{\alpha}}{2}$. The Cauchy-Schwartz inequality is used to develop the following upper bound

$$\|e_u\|^2 \leq \hat{\tau} \int_{t-\hat{\tau}}^t \|\dot{u}(\theta)\|^2 d\theta. \quad (43)$$

Note that using Assumption 4, the inequalities $\int_{t-\tau}^t \|\dot{u}(\theta)\|^2 d\theta \leq \bar{k}^2 \int_{t-(\bar{\tau}+\hat{\tau})}^t \|r(\theta)\|^2 d\theta$ and

$$\int_{t-\hat{\tau}}^t \|\dot{u}(\theta)\|^2 d\theta \leq \bar{k}^2 \int_{t-(\bar{\tau}+\hat{\tau})}^t \|r(\theta)\|^2 d\theta$$

can be obtained. In addition, using the expressions in (22), (23), (31)

and (43), the following inequalities can be obtained

$$-\frac{\omega_2}{4\hat{\tau}} \|e_u\|^2 \geq -\frac{\omega_2}{4} \int_{t-(\bar{\tau}+\hat{\tau})}^t \|\dot{u}(\theta)\|^2 d\theta, \quad (44)$$

$$-\frac{\omega_2 \bar{k}^2 \bar{\alpha}\epsilon_1}{4\omega_1^2 \bar{k}^2} Q_1 \geq -\frac{\omega_2}{4} \int_{t-(\bar{\tau}+\hat{\tau})}^t \|\dot{u}(\theta)\|^2 d\theta, \quad (45)$$

$$-\frac{\omega_2 \bar{k}^3 \alpha (1-\varphi) Q_2}{4(\bar{k}(\bar{\alpha} + \varphi - 1))^2} \geq -\frac{\omega_2}{4} \int_{t-(\bar{\tau}+\hat{\tau})}^t \|\dot{u}(\theta)\|^2 d\theta, \quad (46)$$

$$-\frac{1}{4(\bar{\tau} + \hat{\tau})} Q_3 \geq -\frac{\omega_2}{4} \int_{t-(\bar{\tau}+\hat{\tau})}^t \|\dot{u}(\theta)\|^2 d\theta. \quad (47)$$

By using (44)-(47), (42) can be upper bounded as

$$\begin{aligned} \dot{V} \leq & - \sum_{i=1}^{n-2} \|e_i\|^2 - \left(1 - \frac{\epsilon_2}{2}\right) \|e_{n-1}\|^2 \\ & - \left(\underline{\Lambda} - \left(\frac{\bar{\alpha}}{2\epsilon_1} + \frac{1}{2\epsilon_2}\right)\right) \|e_n\|^2 \\ & - \left(\frac{\omega_2}{4\hat{\tau}} - \bar{\alpha}\epsilon_1\right) \|e_u\|^2 - \frac{\alpha k}{8} \|r\|^2 \\ & - \left(\frac{\alpha k}{8} - \kappa\right) \|r\|^2 + \frac{1}{\alpha k} \rho^2 (\|z\|) \|z\|^2 \\ & + \frac{1}{\alpha k} \zeta_{N_r}^2 + \frac{\bar{\alpha} \|\dot{u}_{\hat{\tau}} - \dot{u}_\tau\|^2}{2} \\ & - \frac{\omega_2 \bar{k}^2 \bar{\alpha}\epsilon_1}{4\omega_1^2 \bar{k}^2} Q_1 - \frac{\omega_2 \bar{k}^3 \alpha (1-\varphi)}{4(\bar{k}(\bar{\alpha} + \varphi - 1))^2} Q_2 \\ & - \frac{1}{4(\bar{\tau} + \hat{\tau})} Q_3. \end{aligned} \quad (48)$$

Note that the Mean Value Theorem can be used to obtain the inequality $\|\dot{u}_{\hat{\tau}} - \dot{u}_\tau\| \leq \|\ddot{u}(\Theta(t, \hat{\tau}))\| |\hat{\tau}|$, where $\Theta(t, \hat{\tau})$ is a point in time between $t-\tau$ and $t-\hat{\tau}$. Furthermore, using the gain conditions in (29), the definition of σ in (20), and the inequality $\|y\| \geq \|z\|$, the following upper bound can be obtained

$$\begin{aligned} \dot{V} \leq & - \left(\frac{\sigma}{2} - \frac{1}{\alpha k} \rho^2 (\|y\|)\right) \|z\|^2 - \frac{\sigma}{2} \|z\|^2 \\ & + \frac{1}{\alpha k} \zeta_{N_r}^2 + \frac{\bar{\alpha} \bar{\tau}^2 \|\ddot{u}(\Theta(t, \hat{\tau}))\|^2}{2} \\ & - \frac{\omega_2 \bar{k}^2 \bar{\alpha}\epsilon_1}{4\omega_1^2 \bar{k}^2} Q_1 - \frac{\omega_2 \bar{k}^3 \alpha (1-\varphi)}{4(\bar{k}(\bar{\alpha} + \varphi - 1))^2} Q_2 \\ & - \frac{1}{4(\bar{\tau} + \hat{\tau})} Q_3. \end{aligned} \quad (49)$$

Provided $y(\eta) \in \mathcal{D} \forall \eta \in [t_0, t]$, then from the definition

of δ in (21), the expression in (49) reduces to

$$\begin{aligned} \dot{V} &\leq -\delta \|y\|^2, \\ \forall \|y\| &\geq \left(\frac{2\zeta_{N_r}^2 + \alpha \underline{k} \bar{\alpha} \bar{\tau}^2 M^2}{2\alpha \underline{k} \delta} \right)^{\frac{1}{2}}. \end{aligned} \quad (50)$$

Using techniques similar to Theorem 4.18 in [49] it can be concluded that y is uniformly ultimately bounded in the sense that $\limsup_{t \rightarrow \infty} \|y(t)\| \leq \sqrt{\frac{\max\{1, \frac{\omega_1}{2}\} (2\zeta_{N_r}^2 + \alpha \underline{k} \bar{\alpha} \bar{\tau}^2 M^2)}{\min\{\frac{1}{2}, \frac{\omega_1}{2}\} 2\alpha \underline{k} \delta}}$ provided $y(t_0) \in \mathcal{S}_{\mathcal{D}}$, where uniformity in initial time can be concluded from the independence of δ and the ultimate bound from t_0 .

Remark 3 *If the system dynamics are such that \tilde{N} is linear in z , then the function ρ can be selected to be a constant, i.e., $\rho(\|z\|) = \bar{\rho}$, $\forall z \in \mathbb{R}^{(n+2)m}$ for some known $\bar{\rho} > 0$. In this case, the last sufficient condition in (29) reduces to*

$$\underline{k} \geq \frac{2\bar{\rho}^2}{\sigma \alpha}, \quad (51)$$

and the result is global in the sense that $\mathcal{D} = \mathcal{S}_{\mathcal{D}} = \mathbb{R}^{(n+2)m+3}$.

Since $e_i, r, e_u \in \mathcal{L}_\infty, i = 1, 2, 3, \dots, n$, from (2), $u \in \mathcal{L}_\infty$. An analysis of the closed-loop system shows that the remaining signals are bounded.

5 Simulation Results

To illustrate performance of the developed controller, numerical simulations were performed on a two-link revolute, direct-drive robot¹¹ with the following dynamics

$$\begin{aligned} \begin{bmatrix} u_{1\tau} \\ u_{2\tau} \end{bmatrix} &= \begin{bmatrix} p_1 + 2p_3c_2 & p_2 + p_3c_2 \\ p_2 + p_3c_2 & p_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} \\ &+ \begin{bmatrix} -p_3s_2\dot{x}_2 & -p_3s_2(\dot{x}_1 + \dot{x}_2) \\ p_3s_2\dot{x}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} \\ &+ \begin{bmatrix} f_{d1} & 0 \\ 0 & f_{d2} \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}, \end{aligned} \quad (52)$$

where $x, \dot{x}, \ddot{x} \in \mathbb{R}^2$. Additive disturbances are applied as $d_1 = 0.2 \sin(0.5t)$ and $d_2 = 0.1 \sin(0.25t)$. Additionally, $p_1 = 3.473 \text{ kg} \cdot \text{m}^2$, $p_2 = 0.196 \text{ kg} \cdot \text{m}^2$, $p_3 = 0.242 \text{ kg} \cdot \text{m}^2$, $p_4 = 0.238 \text{ kg} \cdot \text{m}^2$, $p_5 = 0.146 \text{ kg} \cdot \text{m}^2$, $f_{d1} = 5.3 \text{ Nm} \cdot \text{sec}$, $f_{d2} = 1, 1 \text{ Nm} \cdot \text{sec}$, and s_2, c_2 denote $\sin(x_2)$, and $\cos(x_2)$, respectively.

¹¹ Provided the inertia matrix is known, the dynamics in (52) can be described using (1) [15].

Table 1
RMS errors for time-varying time-delay rates and magnitudes.

	$\tau_i(t)$ (ms)	$\hat{\tau}(t)$ (ms)	RMS Error	
			x_1	x_2
Case 1	$10 \sin(5t) + 10$	15	0.1315°	0.1465°
Case 2	$10 \sin(5t) + 10$	100	0.4774°	0.2212°
Case 3	$60 \sin(t) + 60$	75	0.7479°	0.9928°
Case 4	rand [0, 120]	75	0.7476°	1.0068°

The initial conditions for the system are selected as $x_1, x_2 = 0$. The desired trajectories are selected as

$$\begin{aligned} x_{d1}(t) &= (30 \sin(1.5t) + 20) (1 - e^{-0.01t^3}), \\ x_{d2}(t) &= -(20 \sin(t/2) + 10) (1 - e^{-0.01t^3}). \end{aligned}$$

Several simulation results were obtained using various time-varying delays and different estimated delays, shown in Table 1, to demonstrate performance of the developed continuous robust controller. Cases 1 and 2 use a high-frequency, low-amplitude oscillating delay for different delay upper-bound estimates. Cases 3 use a low-frequency, high-amplitude oscillating delay for known and unknown delay. Case 4 uses random, uniformly distributed delay between 0 and 120 ms for each time instance.

The controller in (9) and (10) is implemented for each case. The control gains are selected for Cases 1 and 2 as $\alpha = \text{diag}\{1, 1\}$, $\Lambda = \text{diag}\{50, 20.5\}$, and $k = \text{diag}\{60, 6\}$, and for Cases 3 and 4 as $\alpha = \text{diag}\{1, 1\}$, $\Lambda = \text{diag}\{23, 8\}$, and $k = \text{diag}\{140, 2.75\}$. The root mean square (RMS) errors obtained for each case are listed in Table 1. By comparing the RMS error for Cases 1 and 2, it is clear that selecting a delay estimate closer to the actual upper bound of the unknown delay yields better tracking performance. Case 3 demonstrates that the performance of the developed controller using a constant estimate of the large unknown delay is comparable to the controller in [11] which uses exact knowledge of the time-varying delay¹². Additionally, the developed controller is reasonably robust even for a constant estimate of the delay when the actual delay is long and time-varying. Although the stability analysis in Section 4 assumes a slowly time-varying input delay, Case 4 shows that the developed controller still provides good tracking performance for a discontinuous, high frequency input delay. Results in Figures 1, 2 and 3 depict the tracking errors, control effort, time-varying delays and estimated delays for Cases 1, 3 and 4, respectively.

¹² The RMS errors for x_1 and x_2 obtained using the controller in [11] for the same delay as Case 3 were 0.7544° and 0.9722° , respectively.

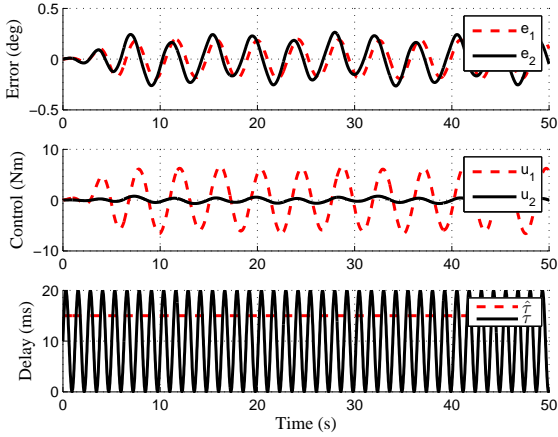


Figure 1. Tracking errors, control effort and time-varying delays vs time for Case 1.

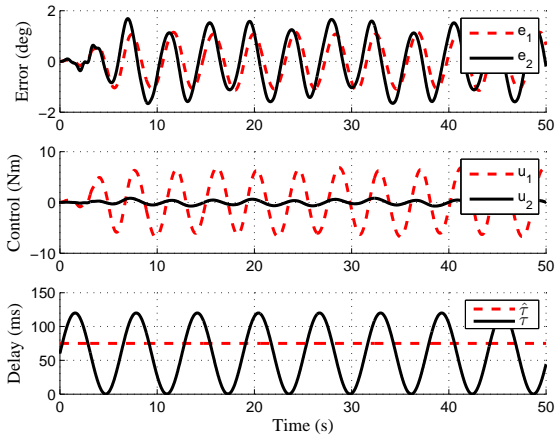


Figure 2. Tracking errors, control effort and time-varying delays vs time for Case 3.

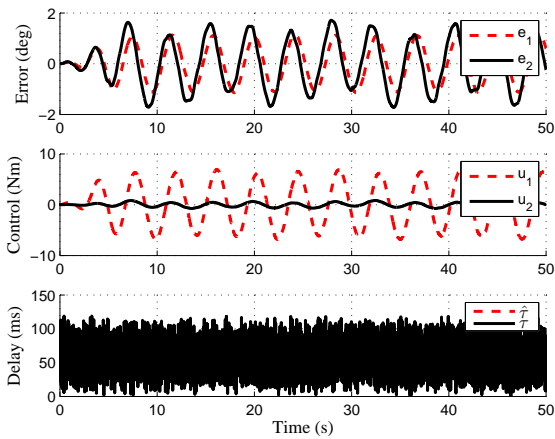


Figure 3. Tracking errors, control effort and time-varying delays vs time for Case 4.

6 Conclusion

Novelty of the controller comes from the fact that a continuous robust controller is developed for a class of uncertain nonlinear systems with additive disturbances subject to uncertain time-varying input time delay. A filtered tracking error signal is designed to facilitate the control design and analysis. A Lyapunov-based analysis is used to prove ultimate boundedness of the error signals through the use of Lyapunov-Krasovskii functionals that are uniquely composed of an integral over the estimated delay range rather than the actual delay range. Simulation results indicate the performance of the controller over a range of time varying delays and estimates. The results even illustrate robustness to random delays up to 120 ms. Improved performance may be obtained by altering the design to allow for a time-varying estimate of the delay. Result such as [50] provides possible insights into estimating/identifying delays in future work.

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