A Switched Adaptive Control Approach to Reduce Sensing Needs in Trajectory Tracking Problems

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***Abstract*— This letter considers an autonomous agent that intermittently acquires state measurements to maintain trajectory tracking performance. The objective is to minimize sensing needs by extending periods of sensor-denied operation. A Lyapunov-based adaptive switched systems approach is developed, where the agent uses the intermittently acquired state measurements to learn the system model. The learned system models are then used during sensor-denied intervals to extend their length while maintaining tracking performance. The design uses a modeling error-dependent bound on the duration of the sensor-denied intervals to progressively reduce sensing needs as the modeling error decreases. The effectiveness of the developed technique is verified in a simulation study.**

***Index Terms*— Switched Systems, parameter estimation, intermittent feedback.**

I. Introduction

AUTONOMOUS agents are frequently deployed in challenging environments such as underwater navigation, subterranean exploration, and military surveillance, where sensor measurements are unavailable or intermittently disrupted [1]. In such scenarios, agents rely on inertial navigation systems and relative sensing techniques for state estimation. However, these methods are inherently prone to error accumulation over time, resulting in drift and degradation in the accuracy of localization [2], [3].

A common technique for inertial navigation is visual or LiDAR-based simultaneous localization and mapping (SLAM). SLAM techniques leverage environmental feature recognition to mitigate localization drift through loop closures, by revisiting previously visited locations [4], [5]. While effective in structured environments, SLAM depends on the presence of distinctive features. In feature-sparse settings, such as underwater domains or planetary surfaces, the performance of SLAM deteriorates and compromises localization accuracy [1].

To address these limitations, recent research has introduced a switched-systems framework [6], [7], where the system transitions between a stabilizable mode with state feedback and an unstable mode without state feedback. Lyapunov-based techniques are used to develop dwell-time conditions [8] that establish the minimum time required in the stable mode and the maximum time allowed in the unstable mode to ensure localization errors remain bounded. However, dwell-time conditions introduced in [6], [7], [9], and [10] are conservative, as they rely on fixed, worst-case error bounds. While recent efforts [11] have analyzed the geometry of the feedback region to allow a larger error bound while still guaranteeing re-entry, the resulting dwell-time conditions remain conservative. This letter develops a learning enabled switched systems framework where a parameter-dependent dwell-time condition is used to increase the maximum allowable time in unstable modes by learning the dynamics of the system. The learned dynamics are used to reduce the growth rate of the error in unstable modes, which allows for extended operations in unstable mode.

The contributions of this letter are two-fold: first, a bound on the maximum allowable time in unstable mode is derived as a function of the parameter estimation error, and second, memory regressor extension (MRE) [12], [13] techniques are used to extend operations in unstable modes while ensuring that parameter estimation errors remain within predefined safety bounds. The developed MRE approach systematically exploits measurements collected during intermittent periods of sensor availability to refine the system model. By leveraging MREs, we improve model accuracy, and the growth of the localization errors is reduced in subsequent sensor-denied intervals. Compared to prior methods [6], [7], [11], the developed framework relaxes conservative dwell-time conditions while preserving Lyapunov-based stability guarantees during operation in sensor-denied intervals.

The developed MRE framework in this letter estimates the parameters of the system, enabling longer operations during sensor-denied periods. To use a refined system model to extend sensor-denied intervals, we derive a modeling error-dependent bound on the time the agent can spend in sensor-denied configuration while maintaining prescribed bounds on tracking performance. The effectiveness of the developed methodology is established through a Lyapunov-based stability analysis and validated via simulation.

II. Problem Formulation

Consider an autonomous agent tasked with tracking a desired trajectory x_d within an environment where only inter-

This research was supported in part by the Air Force Research Laboratory under contract numbers FA8651-24-1-0019 and FA8651-23-1-0006 and the Office of Naval Research under contract number N00014-21-1-2481. Any opinions, findings, or recommendations in this article are those of the author(s), and do not necessarily reflect the views of the sponsoring agencies.

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mittent state feedback is available. A control-affine system describes the dynamics of the agent as

$$\dot{x} = f(t, x) + u + Y(t, x)\theta + d(t, x), \tag{1}$$

where $x \in \mathcal{X} \subset \mathbb{R}^n$ denotes the state, $u : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ denotes the control input, $\theta \in \mathbb{R}^p$ denotes the vector of unknown system parameters, $f : \mathbb{R}_{\geq 0} \times \mathbb{R}^n \to \mathbb{R}^n$ denotes the known drift dynamics, $Y : \mathbb{R}_{\geq 0} \times \mathbb{R}^n \to \mathbb{R}^{n\times p}$ is a known regressor matrix, and $d : \mathbb{R}_{\geq 0} \times \mathbb{R}^n \to \mathbb{R}^n$ represents the unmodeled dynamics and disturbances. For the existence and uniqueness of solutions to (1), the following assumptions are required.

Assumption 1: The functions f and Y are piecewise continuous with respect to t and are locally Lipschitz continuous with respect to x uniformly in t. Furthermore, $f(t, 0) = 0$ and $Y(t, 0) = 0 \ \forall t \in \mathbb{R}_{\geq 0}$.

Under Assumption 1, the regressor Y is bounded on $\mathcal{X}$ by a constant $\bar{Y} > 0$ such that $\|Y(t, x)\| \leq \bar{Y}$, $\forall x \in \mathcal{X}, \forall t \in \mathbb{R}_{\geq 0}$.

Assumption 2: The disturbance d is bounded such that, $\forall (t, x) \in \mathbb{R}_{\geq 0} \times \mathbb{R}^n, \|d(t, x)\| \leq \bar{d}$, where $\bar{d} \in \mathbb{R}_{\geq 0}$.

The control objective is to track a desired trajectory while minimizing the need to measure the system state. In sensor-denied intervals, the controller relies on model-based state estimates. Due to modeling errors induced by the disturbance d and the unknown parameters θ, the state estimation error grows in sensor-denied intervals. The idea in this letter is that since the growth rate of the error depends on the parameter estimation error, the time the agent can spend without access to full state feedback while maintaining prescribed tracking error bounds can be increased as the parameters are learned. In this letter, we develop a scheduling technique that enables the agent to extend its sensor-denied operation time while maintaining stability and tracking performance. In particular, an MRE technique is integrated into a switched-system framework to enhance the sensor-denied operation capabilities.

III. Overview

Let the state estimation error $e_1 : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ be defined as $e_1 := x - \hat{x}$, where $\hat{x} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ is the state estimate. The trajectory tracking error $e_2 : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ is defined as $e_2 := \hat{x} - x_d$, where $x_d : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ denotes the desired trajectory. Let the concatenated error vector be defined as $e := \begin{bmatrix} e_1^\top & e_2^\top \end{bmatrix}^\top : \mathbb{R}_{\geq 0} \to \mathbb{R}^{2n}$. Note that e_1 is only measurable when the agent has access to sensor while e_2 is always measurable. Let $V : \mathbb{R}^{2n} \to \mathbb{R}$ be a candidate Lyapunov function defined as

$$V(e) := \frac{1}{2} e_1^\top e_1 + \frac{1}{2} e_2^\top e_2. \tag{2}$$

In the following, two user-defined bounds on V, an upper bound $V_u \in \mathbb{R}_{>0}$ and a lower bound $V_l \in \mathbb{R}_{>0}$ are used to find an upper bound on the time the agent can spend in a sensor-denied configuration and a lower bound on the time the agent must spend in a sensor-enabled configuration to ensure that $V_l \leq V(e) \leq V_u$ for all t. A Lyapunov-based argument is then used to bound the rate of increase of V when the agent is operating without state feedback and to bound the rate of decrease of V when the agent is operating with state feedback.

To develop the switched systems framework, we assume that the state feedback is available at the initial time t_0^a. Let t_0^u denote the first time instance when the sensors become unavailable. Subsequent sensor availability and unavailability instances are indexed by σ, and denoted by t_σ^a and t_σ^u, respectively. During the interval $t \in [t_\sigma^a, t_\sigma^u)$, sensors are available, while for $t \in \left[t_\sigma^u, t_{\sigma+1}^a\right)$, sensors are not available. Let $\Delta t_\sigma^a := t_\sigma^u - t_\sigma^a$ and $\Delta t_\sigma^u := t_{\sigma+1}^a - t_\sigma^u$ denote the durations of the sensor-available and the sensor-denied intervals, respectively. Let $\mathcal{N}^o \subseteq \mathbb{N}$ denote the number of switches between sensor availability and unavailability.

In this letter, the idea is to leverages state measurements obtained during $[t_\sigma^a, t_\sigma^u)$ to refine estimates of θ and to utilize these improved estimates to progressively extend the permissible dwell-time Δt_σ^u. The extended dwell-time allows the system to track the desired trajectory with fewer sensor-enabled intervals. The following section presents the MRE formulation that enables parameter estimation during $[t_\sigma^a, t_\sigma^u)$.

IV. Memory Regressor Extension

A. Design of Parameter Update Laws

To facilitate the estimation of θ, we express (1) as

$$\mathcal{U}_{f\sigma} = \mathcal{Y}_{f\sigma}\theta + \Xi_{f\sigma}, \quad \sigma \in \mathcal{N}^o, \tag{3}$$

where $\mathcal{U}_{f\sigma} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, $\mathcal{Y}_{f\sigma} : \mathbb{R}_{\geq 0} \to \mathbb{R}^{n\times p}$, and $\Xi_{f\sigma} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ represent the filtered state and control inputs, the filtered regressor matrix, and the filtered residual modeling error, respectively. Several approaches can be employed to compute the filtered signals in (3), including state variable filters [14], regressor filtering [15], dynamic regressor extension and mixing [16], and the windowed integration method [17].

In this work, the windowed integration approach is employed, in which signals are integrated over a finite time window of length $\Delta t > 0$ [17]. Let $\mathcal{I}_u(t) := x(t) - x(t-\Delta t) - \int_{t-\Delta t}^{t} f(\tau, x(\tau)) - u(\tau)\,\mathrm{d}\tau$, $\mathcal{I}_y(t) := \int_{t-\Delta t}^{t} Y(\tau, x(\tau))\,\mathrm{d}\tau$, and $\mathcal{I}_d(t) := \int_{t-\Delta t}^{t} d(\tau, x(\tau))\,\mathrm{d}\tau$ with $t > t_\sigma^a + \Delta t$. Then the signals $\mathcal{U}_{f\sigma} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, $\mathcal{Y}_{f\sigma} : \mathbb{R}_{\geq 0} \to \mathbb{R}^{n\times p}$, and $\Xi_{f\sigma} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ can be defined as

$$\mathcal{U}_{f\sigma}(t) = \begin{cases} \mathcal{I}_u(t) - \mathcal{I}_u(t_\sigma^a), & t \in (t_\sigma^a, t_\sigma^a + \Delta t], \\ \mathcal{I}_u(t) - \mathcal{I}_u(t-\Delta t), & t \in (t_\sigma^a + \Delta t, t_\sigma^u], \\ 0_{n\times 1}, & t \in (t_\sigma^u, t_{\sigma+1}^a], \end{cases} \tag{4}$$

$$\mathcal{Y}_{f\sigma}(t) = \begin{cases} \mathcal{I}_y(t) - \mathcal{I}_y(t_\sigma^a), & t \in (t_\sigma^a, t_\sigma^a + \Delta t], \\ \mathcal{I}_y(t) - \mathcal{I}_y(t-\Delta t), & t \in (t_\sigma^a + \Delta t, t_\sigma^u], \text{ and} \\ 0_{n\times p}, & t \in (t_\sigma^u, t_{\sigma+1}^a], \end{cases} \tag{5}$$

$$\Xi_{f\sigma}(t) = \begin{cases} \mathcal{I}_d(t) - \mathcal{I}_d(t_\sigma^a), & t \in (t_\sigma^a, t_\sigma^a + \Delta t], \\ \mathcal{I}_d(t) - \mathcal{I}_d(t-\Delta t), & t \in (t_\sigma^a + \Delta t, t_\sigma^u], \\ 0_{n\times 1}, & t \in (t_\sigma^u, t_{\sigma+1}^a], \end{cases} \tag{6}$$

where $\Xi_{f\sigma}$ is bounded as $\|\Xi_{f\sigma}(t)\| \leq 2\bar{d}\Delta t$, $t \in (t_\sigma^a, t_\sigma^u]$ using Assumption 2. Once $\mathcal{U}_{f\sigma}$ and $\mathcal{Y}_{f\sigma}$ are obtained, an MRE technique is employed to construct signals $\mathcal{U}_\sigma : \mathbb{R}_{\geq 0} \mapsto \mathbb{R}^p, \mathcal{Y}_\sigma : \mathbb{R}_{\geq 0} \mapsto \mathbb{R}^{p\times p}$, and $\Xi_\sigma : \mathbb{R}_{\geq 0} \mapsto \mathbb{R}^p$, that satisfy $\mathcal{U}_\sigma = \mathcal{Y}_\sigma\theta + \Xi_\sigma$, where $\mathcal{U}_\sigma$ and $\mathcal{Y}_\sigma$ can be computed using measurements of x and u and Ξ_σ is unmeasurable. For example, in concurrent learning (CL) [17], a set of discrete time instances $\{t_i\}_{i=1}^N \subseteq [t_\sigma^a, t_\sigma^u)$, with $N > p$, is selected to store the values of $\mathcal{U}_{f\sigma}$ and $\mathcal{Y}_{f\sigma}$ [17]. Using these time-stamped samples, $\mathcal{U}_\sigma, \mathcal{Y}_\sigma$ and Ξ_σ can be expressed for $t \in [t_\sigma^a, t_\sigma^u)$ as

$$\mathcal{U}_\sigma(t) := \sum_{i=1}^N \frac{\mathcal{Y}_{f\sigma}^\top(t_i(t))\mathcal{U}_{f\sigma}(t_i(t))}{1 + \|\mathcal{Y}_{f\sigma}(t_i(t))\|^2},\ \mathcal{U}_\sigma(t_\sigma^a) = 0_{p\times 1}, \quad (7)$$

$$\mathcal{Y}_\sigma(t) := \sum_{i=1}^N \frac{\mathcal{Y}_{f\sigma}^\top(t_i(t))\mathcal{Y}_{f\sigma}(t_i(t))}{1 + \|\mathcal{Y}_{f\sigma}(t_i(t))\|^2},\ \mathcal{Y}_\sigma(t_\sigma^a) = 0_{p\times p}, \quad (8)$$

$$\Xi_\sigma(t) := \sum_{i=1}^N \frac{\mathcal{Y}_{f\sigma}^\top(t_i(t))\Xi_{f\sigma}(t_i(t))}{1 + \|\mathcal{Y}_{f\sigma}(t_i(t))\|^2},\ \Xi_\sigma(t_\sigma^a) = 0_{p\times 1}. \quad (9)$$

Using bound $\Xi_{f\sigma}$, a bound on Ξ_σ is derived as $\|\Xi_\sigma(t)\| \leq N\bar{d}\Delta t$. Another approach to compute $\mathcal{U}_\sigma, \mathcal{Y}_\sigma$, and Ξ_σ is using exponentially weighted integrals [18], where a forgetting factor $\alpha > 0$ is used to prioritize recent data while retaining informative contributions from earlier measurements. The developed framework can also incorporate other MRE techniques, such as those developed in [19]–[21].

Remark 1: This letter assumes that the measurements and the integrals are exact for the ease of exposition. Measurement errors and numerical integration errors can be easily incorporated in the analysis by adding them to the residual Ξ_σ and bounding it appropriately.

Convergence of the parameter estimation error to a neighborhood of the origin follows if the regressor $\mathcal{Y}_\sigma$ is sufficiently exciting, as formalized in the following assumption.

Assumption 3: The regressor $\mathcal{Y}_\sigma$ is uniformly sufficiently exciting i.e. for all $\sigma \in \mathcal{N}^o$, there exist constants $\lambda_y \in \mathbb{R}_{>0}$ and $T_\sigma \in [t_\sigma^a, t_\sigma^u)$ such that for $T_\sigma \leq t \leq t_{\sigma+1}^a$, and for all initial conditions $e_1(t_\sigma^a), e_2(t_\sigma^a)$, and $\tilde{\theta}(t_\sigma^a)$, $\lambda_{\min}(\mathcal{Y}_\sigma) > \lambda_y$, where $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue of a matrix.

Based on the stability analysis in Section IV-B and under Assumption 3, an adaptive update law is designed as

$$\dot{\hat{\theta}} = \begin{cases} k_\theta \Gamma \left(\mathcal{U}_\sigma(t) - \mathcal{Y}_\sigma(t)\hat{\theta}\right), & t \geq T_\sigma, \\ 0_{p\times 1}, & t < T_\sigma, \end{cases} \quad (10)$$

where $k_\theta > 0$ is a constant scalar, $\Gamma \in \mathbb{R}^{p\times p}$ is a constant positive definite matrix, and $\hat{\theta} \in \mathbb{R}^p$ denotes the estimate of the constant vector θ. Let the parameter estimation error be defined as, $\tilde{\theta} := \theta - \hat{\theta}$. Differentiating $\tilde{\theta}$ and substituting the update law from (10) yields the parameter estimation error dynamics

$$\dot{\tilde{\theta}} = \begin{cases} -k_\theta \Gamma \mathcal{Y}_\sigma(t)\tilde{\theta} - k_\theta \Gamma \Xi_\sigma(t), & t \geq T_\sigma, \\ 0_{p\times 1}, & t < T_\sigma. \end{cases} \quad (11)$$

Note that the update law (10) uses only the measurable signals $\mathcal{U}_\sigma$ and $\mathcal{Y}_\sigma$. The error dynamics (11) is presented only for the purpose of analysis. The next section analyzes the stability of the parameter estimation error system in (11) and establishes uniform ultimate boundedness of its trajectories.

B. Stability Analysis for CL Update Laws

The stability properties of the MRE-based observer developed in (10) are summarized in the following lemma.

Lemma 1: If Assumptions 1–3 are satisfied, then the adaptive update law in (10) ensures that the parameter estimation error $\tilde{\theta}$ is global uniformly ultimately bounded during the sensor-available interval $[t_\sigma^a, t_\sigma^u)$ and satisfies

$$\|\tilde{\theta}(t)\| \leq \sqrt{\frac{\overline{\Gamma}}{\underline{\Gamma}}\left(\|\tilde{\theta}(t_\sigma^a)\|^2 e^{-\rho(t-t_\sigma^a)/\overline{\Gamma}} + \frac{\varpi}{\rho}\left(1 - e^{-\rho(t-t_\sigma^a)/\overline{\Gamma}}\right)\right)}, \quad (12)$$

for all $\sigma \in \mathcal{N}^o$, where $\varpi := 2k_\xi \bar{d}$ and $\rho := \lambda_y$ are constants.

Proof: Consider the candidate Lyapunov function $V_\sigma : \mathbb{R}^p \to \mathbb{R}_{\geq 0}$ defined as $V_\sigma(\tilde{\theta}) := \frac{1}{2}\tilde{\theta}^\top \Gamma^{-1}\tilde{\theta}$, which satisfies the inequality $\underline{\Gamma}\|\theta\|^2 \leq V_\sigma(\tilde{\theta}) \leq \overline{\Gamma}\|\theta\|^2$ for $\sigma \in \mathcal{N}^o$, where the bounds $\underline{\Gamma} := \lambda_{\min}\left(\Gamma^{-1}\right)$ and $\overline{\Gamma} := \lambda_{\max}\left(\Gamma^{-1}\right)$ are positive constants. Taking the Lie derivative of V_σ along the flow of the parameter error dynamics (11) yields

$$\dot{V}_\sigma(t, \tilde{\theta}) = \begin{cases} -k_\theta \tilde{\theta}^\top \mathcal{Y}_\sigma(t)\tilde{\theta} - k_\theta \tilde{\theta}^\top \Xi_\sigma(t), & t \geq T_\sigma, \\ 0, & t < T_\sigma. \end{cases} \quad (13)$$

Using Assumption 2 and Assumption 3, the Lie derivative of the candidate Lyapunov function is bounded by $\dot{V}_\sigma(t, \tilde{\theta}) \leq -k_\theta \lambda_y \|\tilde{\theta}\|^2 + k_\theta k_\xi \bar{d}\|\tilde{\theta}\|$, where $k_\xi \in \mathbb{R}_{>0}$ is a bound on Ξ_σ such that $\|\Xi_\sigma(t)\| \leq k_\xi \bar{d}$, $\forall t \in [t_\sigma^a, t_\sigma^u)$. Applying Young's inequality, the Lie derivative of V_σ can be bounded as

$$\begin{aligned} \dot{V}_\sigma(t, \tilde{\theta}) &\leq -\frac{1}{2}k_\theta \lambda_y \|\tilde{\theta}\|^2 - \|\tilde{\theta}\|\left(\frac{1}{2}k_\theta \lambda_y \|\tilde{\theta}\| - k_\theta k_\xi \bar{d}\right) \\ &\leq -\frac{1}{2}k_\theta \lambda_y \|\tilde{\theta}\|^2, \quad \forall \|\tilde{\theta}\| \geq \frac{\varpi}{\rho} > 0. \end{aligned} \quad (14)$$

Therefore, by [22, Theorem 4.18], the trajectories of (11) are globally uniformly ultimately bounded (GUUB). This GUUB property guarantees that for any initial condition, there exists a finite time t_r, such that the solution enters and remains within the ultimate bound, i.e., $\left\|\tilde{\theta}(t)\right\| \leq \sqrt{\frac{\overline{\Gamma}}{\underline{\Gamma}}\left(\frac{\varpi}{\rho}\right)}$, $\forall t \geq t_r$. Furthermore, by the Comparison Lemma [22, Lemma 3.4], the differential inequality also yields the transient bound in (12) that holds for all $t \in [t_\sigma^a, t_\sigma^u)$. ■

The bound in Lemma 1 only holds for the sensor-available intervals. However, in the interval $[t_\sigma^u, t_{\sigma+1}^a)$ when the agent does not have access to the state, the parameter estimates $\hat{\theta}$ are not updated, i.e., $\dot{\hat{\theta}}(t) = 0$, $\forall t \in [t_\sigma^u, t_{\sigma+1}^a)$. The parameter estimation error thus remains unchanged during the interval $[t_\sigma^u, t_{\sigma+1}^a)$, i.e.,

$$\|\tilde{\theta}(t_{\sigma+1}^a)\| = \|\tilde{\theta}(t_\sigma^u)\|. \quad (15)$$

In the following section, we extend the above analysis to successive intervals by designing dwell-time conditions that ensure that the parameter estimation error remains bounded for all time. In particular, we present the state observer and the controller, which leverage parameter estimates from (10) to enable the agent to operate within the specified error bounds.

V. Controller and Update Laws

A. Control Design

In the absence of continuous state feedback, an observer is required to estimate the state of the agent. This section presents a switching observer along with a stabilizing controller that enables the agent to track a desired trajectory, regardless of sensor availability. Motivated by the observer design for switched systems presented in [6], the state estimates $\hat{x} \in \mathbb{R}^n$ are computed using an observer of the form

$$\dot{\hat{x}}_\sigma = \begin{cases} f(t,\hat{x}) + Y(t,\hat{x})\hat{\theta} + u + v_{r,\sigma}, & \forall t \in [t_\sigma^a, t_\sigma^u), \\ f(t,\hat{x}) + Y(t,\hat{x})\hat{\theta} + u, & \forall t \in [t_\sigma^u, t_{\sigma+1}^a), \end{cases} \tag{16}$$

with the sliding mode term $v_{r,\sigma}$ designed as $v_{r,\sigma} = k_1 e_1 + (\bar{d} + \bar{Y}\tilde{\theta}_\sigma)\,\mathrm{sgn}(e_1)$, where $k_1 \in \mathbb{R}^{n\times n}$ is a positive definite control gain matrix, $\mathrm{sgn}(\cdot)$ is the sign function, and $\tilde{\theta}_\sigma = \|\tilde{\theta}(t_\sigma^u)\|$ is the error bound at start of the sensor-denied interval derived in (12). Note that since the observer is discontinuous, solutions of (16) are interpreted in a generalized sense as Filippov's solutions [23].

For notational brevity, the dependence of all functions on t is omitted hereafter unless needed for clarity. Also, let $f_x := f(t,x)$, $f_{\hat{x}} := f(t,\hat{x})$, $Y_x := Y(t,x)$, $Y_{\hat{x}} := Y(t,\hat{x})$, and $d_x := d(t,x)$. Using the state and parameter estimates from (16) and (10), respectively, a stabilizing controller can be designed as

$$u = \begin{cases} \dot{\bar{x}}_d - f_{\hat{x}} - k_2 e_2 - v_{r,\sigma} - Y_{\hat{x}}\hat{\theta}, & \forall t \in [t_\sigma^a, t_\sigma^u), \\ \dot{\bar{x}}_d - f_{\hat{x}} - k_2 e_2 - Y_{\hat{x}}\hat{\theta}, & \forall t \in [t_\sigma^u, t_{\sigma+1}^a), \end{cases} \tag{17}$$

where $k_2 \in \mathbb{R}^{n\times n}$ is a positive definite tracking error gain matrix. Substituting (1) and (16) into the time derivative of e_1 yields the state estimation error dynamics

$$\dot{e}_1 = \begin{cases} f_x + Y_x\theta + d_x - f_{\hat{x}} - Y_{\hat{x}}\hat{\theta} - v_{r,\sigma}, & \forall t \in [t_\sigma^a, t_\sigma^u), \\ f_x + Y_x\theta + d_x - f_{\hat{x}} - Y_{\hat{x}}\hat{\theta}, & \forall t \in \left[t_\sigma^u, t_{\sigma+1}^a\right). \end{cases} \tag{18}$$

Similarly, substituting (16) into the derivative of e_2 yields the trajectory tracking error dynamics

$$\dot{e}_2 = \begin{cases} f_{\hat{x}} + u + v_{r,\sigma} - \dot{\bar{x}}_d + Y_{\hat{x}}\hat{\theta}, & \forall t \in [t_\sigma^a, t_\sigma^u), \\ f_{\hat{x}} + u - \dot{\bar{x}}_d + Y_{\hat{x}}\hat{\theta}, & \forall t \in \left[t_\sigma^u, t_{\sigma+1}^a\right). \end{cases} \tag{19}$$

Using the controller designed in (17), the error dynamics for e_2 for both intervals can be simplified to

$$\dot{e}_2 = -k_2 e_2, \quad \forall t \in \left[t_\sigma^a, t_{\sigma+1}^a\right). \tag{20}$$

The following section presents the main theoretical results of this letter and analyzes the stability of the switched error system under the controller in (17).

B. Stability Analysis and Dwell-Time Conditions

In this section, the stability of the developed switched observer is analyzed using the controller designed in (17). Using the Lyapunov function introduced in (2) along with the user-defined bounds V_u and V_l and the error dynamics in (18) and (20), this section establishes the dwell-time conditions necessary to ensure the condition $V_l \le V(e(t)) \le V_u$ is satisfied $\forall t$. To facilitate the analysis, let $\mathcal{E} \subset \mathbb{R}^{2n}$ be a compact set defined as $\mathcal{E} := \left\{[e_1^\top, e_2^\top]^\top \in \mathbb{R}^{2n} \mid x, \hat{x} \in \mathcal{X}\right\}$, and let $\eta > 0$ be selected such that $B(0,\eta) \subset \mathcal{E}$, where $B(0,\eta)$ denotes the open ball of radius η around the origin. It can be observed that whenever the errors $\left[e_1^\top, e_2^\top\right]^\top \in \mathcal{E}$, the Lipschitz bounds introduced in Assumption 1 hold. The following theorem computes the dwell-time conditions for sensor-available and sensor-denied intervals.

Theorem 1: Given the error systems in (18) and (20) with initial conditions satisfying $e(t_0^a) \in \mathcal{E}$, the bounds $V_l \le V(e(t)) \le V_u$ hold if Assumptions 1–3 hold, the switching signal satisfies the dwell-time conditions

$$\Delta t_\sigma^a \ge -\frac{1}{\underline{k}_a}\ln\left(\frac{V_l}{V(e(t_\sigma^a))}\right) \text{ and} \tag{21}$$

$$\Delta t_\sigma^u \le \frac{1}{\underline{k}_u}\ln\left(\frac{V_u + \bar{Z}}{V(e(t_\sigma^u)) + \bar{Z}}\right) \tag{22}$$

hold, where $\underline{k}_a, \underline{k}_u, L_1 > 0$ are user-defined gains, V_u is selected such that $V_u < \frac{1}{2}\eta^2$, and $\bar{Z} = \frac{(\bar{d}+\bar{Y}\tilde{\theta}_\sigma)^2}{2L_1\underline{k}_u}$.

Proof: The generalized derivative $\dot{\tilde{V}}$ (see [23, Equation 13]) of the candidate Lyapunov function in (2) along the flow of (18) and (20) is given by

$$\dot{\tilde{V}}(t,e) = \begin{cases} e_1^\top \mathcal{K}\left(f_x - f_{\hat{x}} - v_{r,\sigma} + d_x + Y_x\theta - Y_{\hat{x}}\hat{\theta}\right) \\ \quad + e_2^\top(-k_2 e_2), \quad \forall t \in [t_\sigma^a, t_\sigma^u), \\ e_1^\top\left(f_x - f_{\hat{x}} + d_x + Y_x\theta - Y_{\hat{x}}\hat{\theta}\right) \\ \quad + e_2^\top(-k_2 e_2), \quad \forall t \in \left[t_\sigma^u, t_{\sigma+1}^a\right), \end{cases} \tag{23}$$

where $\mathcal{K}$ denotes Filippov's differential inclusion [23, Definition 2.1].

1) Analysis in the sensor-available Interval:

Consider the interval $t \in [t_\sigma^a, t_\sigma^u)$, such that $e(t_\sigma^a) \in \mathcal{E}$. Since e_1 is measurable, the term $v_{r,\sigma}$ can be implemented. Substituting $v_{r,\sigma}$ into the first case of (23) yields

$$\dot{\tilde{V}}(t,e) \subset e_1^\top\left(f_x + Y_x\theta + d_x - f_{\hat{x}} - Y_{\hat{x}}\hat{\theta} - k_1 e_1\right) - e_1^\top(\bar{d} + \bar{Y}\tilde{\theta}_\sigma)\,\mathrm{SGN}(e_1) - e_2^\top k_2 e_2, \tag{24}$$

where for a given scalar x, the function $\mathrm{SGN}(x) = 1$ if $x > 0$, $\mathrm{SGN}(x) = -1$ if $x < 0$, and $\mathrm{SGN}(x) = [-1,1]$ if $x = 0$. The function is applied element-wise to vector arguments. Since f is Lipschitz continuous uniformly in t, x is known in sensor-available regions, i.e., $Y_{\hat{x}} = Y_x$, we have $\|d(t,x)\| \le \bar{d}$, and $\|Y_x\tilde{\theta}\| \le \bar{Y}\tilde{\theta}_\sigma$, for all $t \in [t_\sigma^a, t_\sigma^u)$ and $x \in \mathcal{X}$. Thus, $\dot{\tilde{V}}$ can be bounded on $[t_\sigma^a, t_\sigma^u) \times \mathcal{E}$ as

$$\dot{\tilde{V}}(t,e) \le (L_f - \lambda_{\min}(k_1))\|e_1\|^2 - \lambda_{\min}(k_2)\|e_2\|^2, \tag{25}$$

where $L_f > 0$ is the Lipschitz constant of f on $\mathcal{X}$, and for a set $A \subset \mathbb{R}$ and a scalar $b \in \mathbb{R}$, the notation $A \le b$ is used to imply that for all $a \in A$, $a < b$. Letting $\underline{k}_1 = -L_f + \lambda_{\min}(k_1)$ and $\underline{k}_2 = -\lambda_{\min}(k_2)$, we can simplify (25) as

$$\dot{\tilde{V}}(t,e) \le -\underline{k}_1\|e_1\|^2 - \underline{k}_2\|e_2\|^2, \tag{26}$$

for all $(t,e) \in [t_\sigma^a, t_\sigma^u) \times \mathcal{E}$. Letting $\underline{k}_a = \min\{\frac{\underline{k}_1}{2}, \frac{\underline{k}_2}{2}\}$, the generalized derivative of the Lyapunov function V is bounded as $\dot{\tilde{V}}(t,e) \le -\underline{k}_a\, V(e), \forall (t,e) \in [t_\sigma^a, t_\sigma^u) \times \mathcal{E}$. Invoking, for example, [24, Theorem 7.2], it can be concluded that the error system in (18) and (20) is strongly locally asymptotically stable in Δt_σ^a. Using the chain rule for generalized derivatives [24, Proposition 4.2] and the comparison lemma [22, Lemma 3.4], the candidate Lyapunov function satisfies

$$V(e(t)) \le V(e(t_\sigma^a))e^{-\underline{k}_a(t - t_\sigma^a)}, \quad \forall t \in [t_\sigma^a, t_\sigma^u). \tag{27}$$

Setting $t = t_\sigma^u$ and imposing the bound $V(e(t_\sigma^u)) \geq V_l$, we obtain

$$V_l \leq V(e(t_\sigma^u)) \leq V(e(t_\sigma^a))e^{-\underline{k}_a(t_\sigma^u - t_\sigma^a)}. \tag{28}$$

Solving for the dwell-time $\Delta t_\sigma^a = t_\sigma^u - t_\sigma^a$, we obtain the dwell-time condition in (21). Thus, the lower bound $V_l \leq V(t_\sigma^u)$ holds provided the dwell-time condition in (21) is satisfied.

2) Analysis during the sensor-denied Interval: For $t \in [t_\sigma^u, t_{\sigma+1}^a)$, the sliding mode term $v_{r,\sigma}$ is not measurable. Thus, using the second case of equation (23), we can express $\dot{V}$ as

$$\dot{V}(t,e) = e_1^\top \left(f_x - f_{\hat{x}} + d_x + Y_x\theta - Y_{\hat{x}}\hat{\theta}\right) + e_2^\top(-k_2 e_2), \quad \forall t \in \left[t_\sigma^u, t_{\sigma+1}^a\right). \tag{29}$$

Note that using (27) and the fact that $e(t_\sigma^a) \in \mathcal{E}$, it can be concluded that $e(t_\sigma^u) \in \text{Int}(\mathcal{E})$, where $\text{Int}(\mathcal{E})$ denotes the interior of $\mathcal{E}$. Let t'_σ be a time instance such that $t'_\sigma = \inf_{t > t_\sigma^u}\{e(t) \notin \mathcal{E}\}$. Since $e(t_\sigma^u) \in \text{Int}(\mathcal{E})$, the interval $[t_\sigma^u, t'_\sigma]$ is of nonzero length. Since $e(t) \in \mathcal{E}$ on the interval $[t_\sigma^u, t'_\sigma]$, using the local Lipschitz continuity of f and Y and using the bound on d, the time-derivative of the candidate Lyapunov function can be bounded on $[t_\sigma^u, t'_\sigma]$ as $\dot{V}(t,e) \leq L_1\|e_1\|^2 + (\bar{d} + \bar{Y}\tilde{\theta}_\sigma)\|e_1\| - \underline{k}_2\|e_2\|^2$, where $L_1 := L_f + L_Y\tilde{\theta}_\sigma$ and $\underline{k}_2 = \lambda_{\min}(k_2)$, where L_y denotes the Lipschitz constant of Y on $\mathcal{X}$. Using the completion of the squares

$$\dot{V}(t,e(t)) \leq \underline{k}_u V(e(t)) + \frac{(\bar{d} + \bar{Y}\tilde{\theta}_\sigma)^2}{2L_1}, \tag{30}$$

where $\underline{k}_u := \min\{\frac{3L_1}{2}, \underline{k}_2\}$. Using the comparison lemma [22, Lemma 3.4], any solution to this differential inequality starting at time t_σ^u satisfies

$$V(e(t)) \leq V(e(t_\sigma^u))e^{\underline{k}_u(t - t_\sigma^u)} + \bar{Z}(e^{\underline{k}_u(t - t_\sigma^u)} - 1), \tag{31}$$

$\forall t \in [t_\sigma^u, t'_\sigma]$. Under the dwell-time condition in (22), we obtain the bound

$$V_u \geq V(e(t_\sigma^u))e^{\underline{k}\Delta t_\sigma^u} + \frac{(\bar{d} + \bar{Y}\tilde{\theta}_\sigma)^2}{2L_1\underline{k}}(e^{\underline{k}\Delta t_\sigma^u} - 1). \tag{32}$$

Since V_u is selected to ensure that $V(e) \leq V_u \implies e \in \mathcal{E}$, we conclude that $t'_\sigma \geq t_{\sigma+1}^a$, and as a result, for all $t \in [t_\sigma^u, t_{\sigma+1}^a]$, $V(e(t)) \leq V_u$ and $e(t) \in \mathcal{E}$. In particular, $V(e(t_{\sigma+1}^a)) \leq V_u$ and $e(t_{\sigma+1}^a) \in \mathcal{E}$. Since $e(t_0^a) \in \mathcal{E}$, an inductive argument can be used to conclude that $\forall \sigma \in \mathcal{N}^o$, $e(t_\sigma^a) \in \mathcal{E}$ and for all $t \geq t_0^a$, $V_l \leq V(e(t)) \leq V_u$. ■

The following theorem presents the main result of this letter.

Theorem 2: If the conditions of Lemmas 1 and Theorem 1 are satisfied, then the sequence of dwell times $\{\Delta t_i^u\}_{i=1,\ldots,\bar{\mathcal{N}}-1}$ satisfies the inequalities

$$\Delta t_1^u < \Delta t_2^u < \cdots < \Delta t_{\bar{\mathcal{N}}-1}^u, \tag{33}$$

where $\bar{\mathcal{N}}$ is the first switching index σ such that $\|\tilde{\theta}(t_\sigma^a)\| \leq \sqrt{\frac{\varpi}{\rho}}$ or specifically $t_{\bar{\mathcal{N}}}^u > t_r$.

Proof: From Lemma 1, after the first k sensor-available intervals

$$\|\tilde{\theta}(t_{\sigma+k}^u)\| \leq \sqrt{\alpha^{k+1}\|\tilde{\theta}(t_\sigma^a)\|^2 \prod_{j=0}^{k} e^{-\frac{\rho}{\overline{\Gamma}}\Delta t_{\sigma+j}} + \gamma \cdot \Phi_k}, \tag{34}$$

where $\Phi_k := \sum_{j=0}^{k} \alpha^{k-j} \prod_{\ell=j+1}^{k} e^{-\frac{\rho}{\overline{\Gamma}}\Delta t_{\sigma+\ell}}\left(1 - e^{-\frac{\rho}{\overline{\Gamma}}\Delta t_{\sigma+j}}\right)$, $\alpha := \frac{\overline{\Gamma}}{\underline{\Gamma}}$, and $\gamma := \frac{\varpi}{\rho}$. After a finite time t_r, the estimation error $\tilde{\theta}$ remains confined within a ball of radius $\sqrt{\frac{\overline{\Gamma}}{\underline{\Gamma}}\frac{\varpi}{\rho}}$. Letting $\bar{\mathcal{N}} \subset \mathcal{N}^o$ denote the subset of switching indices such that $t_\sigma^u < t_r$, the parameter estimation errors satisfy

$$\|\tilde{\theta}(t_{\sigma+k}^u)\| < \|\tilde{\theta}(t_\sigma^u)\|, \quad \forall \sigma \in \bar{\mathcal{N}}. \tag{35}$$

By applying (35) across the $\bar{\mathcal{N}}$ intervals and provided (22) in Theorem 1 holds, the sequence of dwell times satisfy (33). ■

From Theorem 2, we conclude that the sensor-denied interval is progressively extended, up to a limit dictated by the ultimate bound on $\tilde{\theta}$. The simulation results are presented in the next section.

VI. Simulation

Consider a dynamical system of the form in (1), where $x = [x_1; x_2] \in \mathbb{R}^2$, $f(t,x) = [x_2; x_1]$, $Y(t,x) = [0, 0; -x_1, -x_1^3]$, and $\theta = [1; 0.5]$. The dynamics are subjected to a norm-bounded disturbance $d(t,x) = [0.1\sin(2x_1); 0]$. At $t = 0$, the full state signal is available and the initial conditions are selected as $x(0) = [-1; 1]$, $\hat{x}(0) = 0_{2\times1}$, and $\hat{\theta}(0) = 0_{2\times1}$. The parameter estimation law employs a CL strategy that maintains a history stack of $N = 15$ data points during Δt_σ^a. The adaptation gain is set to $\Gamma = I_2$ with $k_\theta = 0.5$. Integrals in (4)–(6) are computed over a fixed window $\Delta t = 0.25$ s, and a history stack entry is added only if the minimum eigenvalue of the information matrix increases by more than $\lambda_y = 0.04$. The delay differential equations governing the CL dynamics are integrated using the MATLAB `dde23` solver. At $t_0^u = 2s$, a switching condition is triggered as the state becomes unavailable and the updated parameter estimates are used to compute Δt_0^u using (22). During $[t_0^u, t_1^a)$, the system dynamics are propagated using the MATLAB `ODE45` solver based on updated parameter estimates. The developed framework is compared with results from [6] as shown in Figure 4.

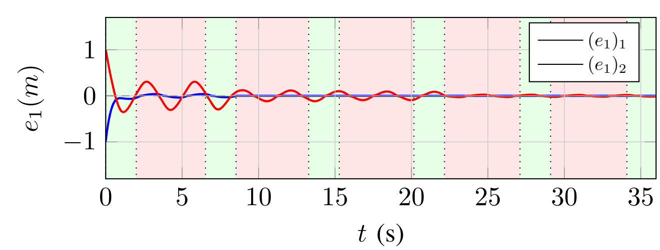

Fig. 1: Trajectories of the state estimation error $e_1(t)$ over time. Time intervals with sensor availability are highlighted in green, while sensor-denied intervals are shown in red.

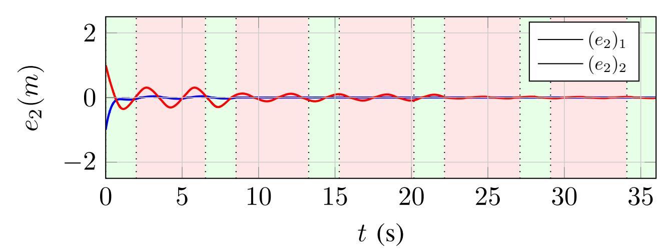

Fig. 2: Trajectories of trajectory tracking error $e_2(t)$ over time.

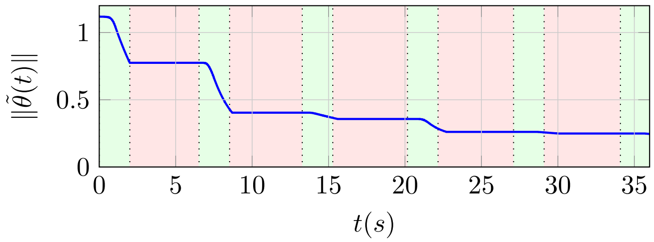

Fig. 3: Trajectory of $\|\tilde{\theta}(t)\|$ over time.

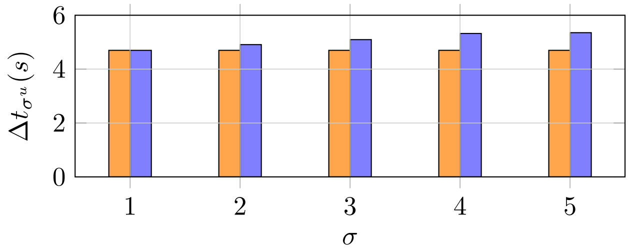

Fig. 4: Dwell time over five switches in sensor-denied intervals, with orange showing Ref. [5] and blue showing the proposed approach.

VII. Discussion

Figures 1-2 show that the control law (17) drives the tracking errors to a small neighborhood of the origin. With the MRE-based update (10), the decaying parameter-error norm $\|\tilde{\theta}(t)\|$ in Fig. 3 indicates effective learning under intermittent sensing. Figure 4 further shows longer admissible dwell times than conservative switched-system bounds (e.g., [6]), reducing unnecessary switching.

A limitation of the present framework is chattering induced by the sliding-mode term. The analysis further assumes a parametric disturbance model, and sensitivity to measurement noise increases with higher gains and shorter integral windows. These issues can be mitigated with standard techniques e.g., boundary layers or higher-order sliding to reduce chattering, disturbance observers to relax modeling assumptions, and filtering with conservative gain/window selections to temper noise [12], [13].

VIII. Conclusion

In intermittent sensor applications, error bounds impose strict dwell-time conditions on system operation. This letter addresses this limitation by integrating a switched-systems framework with MRE parameter estimation, which leverages sensor-available intervals to estimate unknown parameters and compensates unmodeled dynamics. As a result, the growth of state estimation errors is controlled, enabling prolonged operation in sensor-denied intervals. Future work will focus on experimental validation on robotic platforms operating in constrained environments with intermittent sensing.

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