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Cooperative Manipulation of an Unknown Payload with Concurrent Mass and Drag Force Estimation

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Abstract— **In this paper, we consider a team of robots that cooperatively transport a payload with an unknown mass in the presence of unknown drag forces. We develop a concurrent learning based adaptive control algorithm that estimates the drag forces and the unknown mass and drives the agents and the payload to a common desired velocity. The algorithm also regulates the contact forces on the payload. We prove that the estimated parameters, including the mass of the payload, converge to their true values. We validate the effectiveness of the proposed algorithm using two simulation examples.**

I. INTRODUCTION

Cooperative manipulation has been an active area of research for decades due to the complexity of the problem involved (see e.g., [1]–[4]). Multiple robots can be employed together to increase the capability of each individual robot, which enables better manipulation and control of heavier payloads for various applications. Possible applications of such multi-robot systems include disaster response, transport, manufacturing, search and rescue operations, and construction, among others.

Literature on cooperative manipulation is extensive, ranging from cooperative assembly robots in manufacturing [2], [5], ground manipulation with mobile robots [6], multi-rotor based aerial manipulation [1], and marine applications using autonomous tugboats [7]. Recently, the authors in [8] develop a framework for control and estimation of an unknown payload and obstacle avoidance for cooperative aerial manipulation. The authors in [3] develop a decentralized model reference adaptive controller to transport a rigid payload in $\mathbb{R}^2$ or $\mathbb{R}^3$ by controlling the spatial velocity of the payload. Reference [9] develop a decentralized passive force control strategy for collaborative manipulation using micro-aerial vehicles based on master-slave methodology. The authors in [6] propose a distributed approach to estimate the internal properties of the payload (possibly large) using noisy measurements of velocity and contact forces applied to the payload. Reference [7] develop an adaptive force/torque control strategy that allows a swarm of autonomous tugboats to cooperatively move a heavier object on water to compensate for model uncertainties associated with the drag coefficient. Cooperative manipulation can also be formulated using the concept of virtual structure, where a fixed geometric relationship between the agents and the payload is devised to accomplish different maneuvers (see e.g., [10]).

In this paper, we extend our previous work [11] to address the problem of force control in cooperative manipulation in the presence of unknown drag forces and unknown payload mass in either $\mathbb{R}^2$ or $\mathbb{R}^3$. Our control objective is that all the agents and the payload converge to a constant velocity and the contact force is regulated to a desired set-point. Since there are multiple unknown parameters, such as the mass and the drag coefficients, and the desired velocity does not possess the property of persistency of excitation (PE), we employ the recently developed concurrent learning (CL) technique [12], [13] to address the parameter estimation problem. CL-based algorithms leverage transient data to estimate unknown parameters with a relaxed excitation condition in the regressor. Specifically, we develop an adaptive control law that consists of a CL update law and a stabilizing update law. The proposed algorithm simultaneously estimates the coefficients of the drag forces acting on the robots and the payload and the unknown mass of the payload online. We establish stability of the closed-loop system using Lyapunov theory. Using two numerical simulation, we demonstrate that the developed algorithm achieves the control objective and parameter convergence without PE.

The contribution of this paper is the development, application, stability analysis of a CL based controller for cooperative manipulation and demonstrating its benefit, i.e., parameter convergence without requiring a PE velocity profile. Parameter convergence typically leads to improved transient performance [14] and once the parameters are estimated, the estimates can be used as initial guesses in future executions of the control task, resulting in further improvements in transient performance. We consider only translational motion to illustrate the CL based control design process in its basic form. Extension to simultaneous attitude and position control, e.g., [15], will be pursued.

The rest of this paper is organized as follows. We formulate the cooperative payload transport problem in Section II. In Section III, we develop a CL based adaptive control law to achieve the force regulation of the payload and velocity convergence of the agents. In Section IV, we analyze the stability of the system using Lyapunov theory. Two numerical examples are presented in Section V to demonstrate the performance of our control law. Conclusions and future work are discussed in Section VI.

II. PROBLEM FORMULATION

A. Dynamic Model

Consider N agents holding a common load as shown in Fig. 1 ($N = 3$). Each agent is a robot with a rigid link extension. Agent i is attached to the load at the point a_i. Let $x_i \in \mathbb{R}^2$ or $\mathbb{R}^3$ be the position of the end-effector of agent i in the inertial frame and $r_i \in \mathbb{R}^3$ be a fixed vector in the body frame of the load. Initially, $x_i(0) = a_i(0) = x_c(0) + r_i$, where

$x_c \in \mathbb{R}^2$ or $\mathbb{R}^3$ is the position of the center of mass of the load in the inertial frame. Fig. 1 also shows the coordinate system defined to derive the kinematics of the system. Σ_I is the world fixed inertial frame and $O_{c,i}$ is the body-fixed frame attached to each agent i.

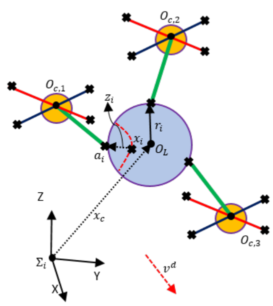

Fig. 1: Three robots transport a common load. Note that a_i is the initial position of agent i in the inertial frame. As the agents move, the payload is deformed, the new position of the agent is x_i and the deformation is approximated by $z_i = x_i - a_i$.

We assume that the payload is a rigid object surrounded by elastic or deformable materials, such as bumpers and springs. Successful experimental results using manipulators in [2] have validated this assumption for payloads like a soccer ball. When there is relative motion between an agent and the payload, the surrounding material will be deformed. We use a mass spring model to model the contact force generated due to the deformation. Specifically, we define a_i as

$$a_i(t) := x_c(t) + r_i, \tag{1}$$

which is the position of the ith agent if there is no deformation. Note that a_i satisfies

$$\dot{a}_i = \dot{x}_c. \tag{2}$$

We approximate the deformation as

$$z_i = x_i - a_i, \quad \forall i = 1, \cdots, N, \tag{3}$$

and assume that the contact force f_i between agent i and the payload is given by the gradient of a positive-definite and *strictly convex* potential function $P_i : \mathbb{R}^3 \to \mathbb{R}_{\geq 0}$, i.e.,

$$f_i(z_i) = \nabla P_i(z_i). \tag{4}$$

We further assume that P_i satisfies the following constraints:

$$P_i(z_i) = 0 \iff z_i = 0, \tag{5}$$
$$\nabla P_i(z_i) = 0 \iff z_i = 0. \tag{6}$$

The strict convexity assumption is satisfied by the linear spring potential model $P_i(z_i) = b_i\|z_i\|^2$, $b_i > 0$, and certain classes of nonlinear models, such as $P_i(z_i) = b_i\|z_i\|^4$.

The dynamics of the load is given by

$$M_c\ddot{x}_c = \sum_{i=1}^{N} f_i(z_i) - M_c g e_3 - F_d, \tag{7}$$

where M_c is the mass of the load, g is the gravitational constant, e_3 is the unit vector $\begin{bmatrix}0 & 0 & 1\end{bmatrix}^T$, and F_d is a constant disturbance acting on the payload. In (7), we approximate the payload dynamics as a rigid body. This is valid when the deformations are small. The translational dynamics of the N agents are given by

$$m_i\ddot{x}_i = F_i - f_i(z_i) - m_i g e_3 - C_i\|\dot{x}_i\|\dot{x}_i, \quad \forall i = 1, ..., N, \tag{8}$$

where m_i is the mass of the ith agent, F_i is the force applied to agent i, f_i is the contact force to agent i and $C_i \in \mathbb{R}^{3\times3}$ is the drag coefficient matrix for agent i. The $C_i\|\dot{x}_i\|\dot{x}_i$ term is a standard drag force model [16], where C_i is a diagonal matrix (see e.g., [16]) given by

$$\text{diag}\{C_i^x, C_i^y, C_i^z\} = \begin{bmatrix} C_i^x & 0 & 0 \\ 0 & C_i^y & 0 \\ 0 & 0 & C_i^z \end{bmatrix}.$$

B. Control Objective

In our previous work [11] and [17], the disturbances F_d and $C_i\|\dot{x}_i\|\dot{x}_i$ were neglected. In this paper, we assume that the load has an unknown mass M_c and that unknown disturbances F_d and $C_i\|\dot{x}_i\|\dot{x}_i$ act on the payload and agent i, $\forall i = 1, \ldots, N$, respectively. Our control objective is to design F_i in (8) such that all the agents and the payload converge to a constant velocity v^d and the contact force f_i is regulated to a set-point f_i^d. Motivated by results in [12], [13], [18]–[20], a concurrent learning adaptive controller is developed to achieve the stated objective.

III. Control Design

If the velocity of the payload converges to v^d, the sum of the contact forces f_i would satisfy

$$\sum_{i=1}^{N} f_i = \underbrace{M_c g e_3 + F_d}_{F_c}. \tag{9}$$

Assuming each agent experiences an equal contact force, we choose the set-points for the individual contact forces as

$$f_i^d = \frac{F_c}{N}. \tag{10}$$

Using the estimates $\hat{M}_c$ and $\hat{F}_d$ for the mass and the constant drag, respectively, the set-points can be estimated as

$$\hat{f}_i^d = \frac{1}{N} Y_f \hat{\theta}_c, \tag{11}$$

where $\hat{\theta}_c = \begin{bmatrix}\hat{F}_d^x, & \hat{F}_d^y, & \hat{F}_d^z, & \hat{M}_c\end{bmatrix}^T$ and $Y_f = \begin{bmatrix}\mathrm{I}_3, & g e_3\end{bmatrix}$, where I_n denotes the $n \times n$ identity matrix.

We propose to design the controller F_i as

$$F_i = -K_i(\dot{x}_i - v^d) + \hat{f}_i^d + m_i g e_3 + \text{diag}\{\hat{C}_i\}\|\dot{x}_i\|\dot{x}_i, \tag{12}$$

where $K_i = K_i^T > 0$. The first term in (12) is the feedback term to ensure $\dot{x}_i \to v^d$, and the second term is the feed-forward term that regulates $f_i \to \hat{f}_i^d$. The last two terms are introduced to cancel the gravity and drag force acting on the agents. We next design a drag force estimation law for each agent to drive $\hat{C}_i \to C_i$.

A. Drag Coefficient Estimation for the Agents

We rewrite the dynamics for each agent as:

$$\ddot{x}_i = \frac{1}{m_i}\left(F_i - f_i(z_i)\right) - Y_i\theta_i, \tag{13}$$

where

$$Y_i = \left[\text{diag}\left\{\frac{1}{m_i}\|\dot{x}_i\|\dot{x}_i\right\}\right] \quad \text{and} \quad \theta_i := \left[C_i^x, C_i^y, C_i^z\right]^T.$$

We propose the following update law to learn the drag force coefficient θ_i

$$\dot{\hat{\theta}}_i = -\mu_i\Gamma_i\Phi_i + \dot{\hat{\theta}}_i^{cl}, \tag{14}$$

where $\mu_i > 0$, $\Gamma_i \in \mathbb{R}^{3\times 3}$ is the learning gain computed using (20), $\dot{\hat{\theta}}_i^{cl}$ is based on the concurrent learning update law in (19) and

$$\Phi_i = \|\dot{x}_i\|\left[\xi_i^1\dot{x}_i^1 \quad \xi_i^2\dot{x}_i^2 \quad \xi_i^3\dot{x}_i^3\right]^T, \tag{15}$$

where ξ_i^j is the jth element of ξ_i and $\xi_i := \dot{x}_i - v^d$.

We next briefly explain the construction of the concurrent learning update law $\dot{\hat{\theta}}_i^{cl}$. Interested readers are referred to [21] for more details.Integrating (13) over the interval $[t-\tau, t]$ for some constant $\tau \in \mathbb{R}_{>0}$,

$$\underbrace{x_i(t) - x_i(t-\tau)}_{P_i(t)} = \underbrace{\int_{t-\tau}^{t} f_o\left(F_i(\gamma), z_i(\gamma)\right)\mathrm{d}\gamma}_{H_i(t)} + \underbrace{\int_{t-\tau}^{t} Y_i(\gamma)\,\mathrm{d}\gamma}_{G_i(t)}\,\theta_i, \tag{16}$$

where $f_o(F_i, z_i) = \frac{1}{m_i}\left(F_i - f_i(z_i)\right)$.

For ease of exposition, it is assumed that a history stack, i.e., a set of ordered pairs $\{(P_{k,i}, H_{k,i}, G_{k,i})\}_{k=1}^M$ such that

$$P_{k,i} = H_{k,i} + \theta_i^T G_{k,i}, \; \forall k \in \{1, \cdots, M\}, \tag{17}$$

is available a priori. A history stack $\{(P_{k,i}, H_{k,i}, G_{k,i})\}_{k=1}^M$ is called *full rank* if there exists a constant $\underline{c_i} \in \mathbb{R}$ such that

$$0 < \underline{c_i} < \lambda_{\min}\{\mathscr{G}_i\}, \tag{18}$$

where the matrix $\mathscr{G}_i \in \mathbb{R}^{3\times 3}$ is defined as $\mathscr{G}_i := \sum_{k=1}^M G_{k,i}G_{k,i}^T$. To select the data points in $\mathscr{G}_i$, a singular value maximization algorithm can be used [21]. If the condition in (18) is not satisfied, i.e., the matrix $\mathscr{G}_i$ is not full rank, then data is added to the history stack $\mathscr{G}_i$ until $\underline{c_i} > 0$. Once (18) is satisfied, then a data point is added to $\mathscr{G}_i$ only if a predefined amount of time has passed since the last change and if it increases the minimum singular value of $\mathscr{G}_i$. Therefore, although $\mathscr{G}_i$ can be discontinuous, it is always piecewise continuous with a lower-bounded dwell time.

The concurrent learning update law is then given by

$$\dot{\hat{\theta}}_i^{cl} = k_i\Gamma_i\sum_{k=1}^M G_{k,i}\left(P_{k,i} - H_{k,i} - \hat{\theta}_i^T G_{k,i}\right)^T, \tag{19}$$

where $k_i \in \mathbb{R}_{>0}$ is a constant adaptation gain, and $\Gamma_i \in \mathbb{R}^{3\times 3}$ is the least-squares gain updated using the update law

$$\dot{\Gamma}_i = \beta_i\Gamma_i - k_i\Gamma_i\frac{1}{1+\alpha_i\|\mathscr{G}_i\|}\mathscr{G}_i\Gamma_i. \tag{20}$$

in which α_i, $\beta_i \in \mathbb{R}_{>0}$. Using arguments similar to [22, Corollary 4.3.2], it can be shown that provided $\lambda_{\min}\left\{\Gamma_i^{-1}(0)\right\} > 0$, the least squares gain matrix satisfies $\underline{\Gamma_i}\mathrm{I}_3 \le \Gamma_i(t) \le \overline{\Gamma_i}\mathrm{I}_3$, where $\underline{\Gamma_i}$ and $\overline{\Gamma_i}$ are positive constants.

B. Mass and Disturbance Estimation for the Payload

We rewrite the payload dynamics (7) as:

$$\begin{bmatrix}\sum_{i=1}^N f_i^x \\ \sum_{i=1}^N f_i^y \\ \sum_{i=1}^N f_i^z\end{bmatrix} = \underbrace{\begin{bmatrix}1 & 0 & 0 & \ddot{x}_{c,x} \\ 0 & 1 & 0 & \ddot{x}_{c,y} \\ 0 & 0 & 1 & \ddot{x}_{c,z}+g\end{bmatrix}}_{Y_C}\underbrace{\begin{bmatrix}F_d^x \\ F_d^y \\ F_d^z \\ M_c\end{bmatrix}}_{\theta_c}. \tag{21}$$

We propose the following update for the payload parameters $\hat{\theta}_c$

$$\dot{\hat{\theta}}_c = -\mu_c\Gamma_c\left(\frac{1}{N}Y_f^T\sum_{i=1}^N \xi_i\right) + \dot{\hat{\theta}}_c^{cl} \tag{22}$$

where $\mu_c > 0$, $\Gamma_c \in \mathbb{R}^{4\times 4}$ is the learning gain computed using (25), and $\dot{\hat{\theta}}_c^{cl}$ is based on the concurrent learning update law developed in (24). Integrating (21) over the interval $[t-\tau, t]$ for some constant $\tau \in \mathbb{R}_{>0}$ yields

$$\underbrace{\int_{t-\tau}^{t}\sum_{i=1}^N f_i(\gamma)\,\mathrm{d}\gamma}_{P_c(t)} = \underbrace{\int_{t-\tau}^{t} Y_c(\gamma)\,\mathrm{d}\gamma}_{G_c(t)}\,\theta_c. \tag{23}$$

The concurrent learning update law to estimate the unknown parameters for the payload is then given by

$$\dot{\hat{\theta}}_c^{cl} = k_c\Gamma_c\sum_{k=1}^M G_{k,c}\left(P_{k,c} - \hat{\theta}_c^T G_{k,c}\right)^T, \tag{24}$$

where $k_c \in \mathbb{R}_{>0}$ is a constant adaptation gain, $\Gamma_c \in \mathbb{R}^{4\times 4}$ is the least-squares gain updated using the update law

$$\dot{\Gamma}_c = \beta_c\Gamma_c - k_c\Gamma_c\frac{1}{1+\alpha_c\|\mathscr{G}_c\|}\mathscr{G}_c\Gamma_c, \tag{25}$$

in which $\alpha_c, \beta_c \in \mathbb{R}_{>0}$, and the matrix $\mathscr{G}_c \in \mathbb{R}^{4\times 4}$ is defined as $\mathscr{G}_c := \sum_{k=1}^M G_{k,c}G_{k,c}^T$.

IV. Stability Analysis

The dynamics (8) with the proposed control (12) takes the following form:

$$m_i\dot{\xi}_i = -K_i\xi_i + \frac{1}{N}Y_f\hat{\theta}_c - f_i - \text{diag}(\tilde{\theta}_i)\|\xi_i + v^d\|(\xi_i + v^d), \tag{26}$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$. We let $\xi_c = \dot{x}_c - v^d$ and obtain the payload dynamics as

$$M_c\dot{\xi}_c = \sum_{i=1}^N f_i - M_c g e_3 - F_d. \tag{27}$$

We further let $\tilde{z}_i = z_i - z_i^d$, $\forall i = 1, \cdots, N$ and $\tilde{\theta}_c = \theta_c - \hat{\theta}_c$. The desired equilibrium of the closed-loop system (26) and (27) with the update laws (14) and (22) is given by

$$\mathcal{E}^* = \Bigg\{ \left(\{\xi_i\}_{i=1}^N, \left\{\tilde{\theta}_i\right\}_{i=1}^N, \tilde{\theta}_c, \{\tilde{z}_i\}_{i=1}^N, \xi_c \right) \Big| \xi_i = 0, \\ \tilde{\theta}_i = 0, \tilde{z}_i = 0,\ \forall i = 1, \cdots, N,\ \tilde{\theta}_c = 0,\ \xi_c = 0 \Bigg\}. \tag{28}$$

Theorem 1 below establishes the asymptotic stability of $\mathcal{E}^*$. Note that convergence to $\mathcal{E}^*$ means that $\xi_i \to 0$ and $\xi_c \to 0$, which indicate the velocities of the payload and the agents converge to v^d. Similarly $\tilde{\theta}_i \to 0$ and $\tilde{\theta}_c \to 0$ indicate the estimates of the drag coefficients, the payload mass and the disturbance converge to the true values. Also $\tilde{z}_i \to 0$ ensures that $f_i \to f_i^d$ which means that the contact forces are regulated. The proof of Theorem 1 relies on the assumption that $\mathscr{G}_i$ and $\mathscr{G}_c$ are both full rank to achieve parameter convergence.

Assumption 1: There exists a $T^* > \tau$ such that over the time interval $[0, T^*)$, the trajectories of the agents and the payload provide enough information for $\mathscr{G}_i$ and $\mathscr{G}_c$ to become full rank. ■

A nonzero velocity in all three directions is required during the transient response (over $[0, T^*]$, $T^* > \tau$) for $\mathscr{G}_i$ and $\mathscr{G}_c$ to be full rank. The following Lyapunov analysis shows that even in the absence of motion in all three directions, the adaptive controller results in Lyapunov stability of the desired equilibrium point; however, asymptotic stability is only guaranteed provided T^* in Assumption 1 exists.

Theorem 1: The control law (12), with the update laws (14) and (22), ensures that the desired equilibrium $\mathcal{E}^*$ in (28) is globally asymptotically stable. ■

Proof: Consider the energy-motivated positive definite candidate Lyapunov function [23, Chap 8] $\left(\{\xi_i\}_{i=1}^N, \xi_c, \{\tilde{z}_i\}_{i=1}^N\right) \mapsto V_1\left(\{\xi_i\}_{i=1}^N, \xi_c, \{\tilde{z}_i\}_{i=1}^N\right)$:

$$V_1 = \sum_{i=1}^N \left[P_i(z_i) - P_i(z_i^d) - (f_i^d)^T (z_i - z_i^d) \right] \\ + \frac{1}{2}\left(\sum_{i=1}^N \xi_i^T m_i \xi_i + \xi_c^T M_c \xi_c \right). \tag{29}$$

Note that because of the strict convexity of $P(z_i)$, the first part of V_1 is positive definite and proper and has a unique global minimum at $z_i = z_i^d$ [24, Proposition 2].

From (2) and (3), the kinematics of z_i is given by

$$\dot{z}_i = \dot{x}_i - \dot{a}_i = \dot{x}_i - \dot{x}_c = \xi_i - \xi_c. \tag{30}$$

The time derivative of V_1 yields

$$\dot{V}_1 = \sum_{i=1}^N (f_i - f_i^d)^T \dot{z}_i + \sum_{i=1}^N \xi_i^T m_i \ddot{x}_i + \xi_c^T M_c \ddot{x}_c. \tag{31}$$

We rewrite (31) from (7), (8), (9), (30) and (12) as

$$\dot{V}_1 = -\sum_{i=1}^N \xi_i^T K_i \xi_i - \sum_{i=1}^N \xi_i^T \frac{1}{N} Y_f \tilde{\theta}_c - \sum_{i=1}^N \Phi_i^T \tilde{\theta}_i. \tag{32}$$

Consider another positive definite candidate Lyapunov function $\left(\left\{\tilde{\theta}_i\right\}_{i=1}^N, \tilde{\theta}_c, t\right) \mapsto V_2\left(\left\{\tilde{\theta}_i\right\}_{i=1}^N, \tilde{\theta}_c, t\right)$:

$$V_2 = \frac{1}{2\mu_c} \tilde{\theta}_c^T \Gamma_c^{-1} \tilde{\theta}_c + \sum_{i=1}^N \frac{1}{2\mu_i} \tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i. \tag{33}$$

Using (14) and (22) together with (19) and (24), we obtain

$$\dot{V}_2 = \tilde{\theta}_c^T \left(\frac{1}{N} Y_f^T \sum_{i=1}^N \xi_i \right) - \frac{k_c}{2\mu_c} \tilde{\theta}_c^T \mathscr{G}_c \tilde{\theta}_c - \frac{\beta_c}{2\mu_c} \tilde{\theta}_c^T \Gamma_c^{-1} \tilde{\theta}_c \\ + \sum_{i=1}^N \tilde{\theta}_i^T \Phi_i - \sum_{i=1}^N \frac{k_i}{2\mu_i} \tilde{\theta}_i^T \mathscr{G}_i \tilde{\theta}_i - \sum_{i=1}^N \frac{\beta_i}{2\mu_i} \tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i, \tag{34}$$

where we have used the identity $\dot{\Gamma}^{-1} = -\Gamma^{-1} \dot{\Gamma} \Gamma^{-1}$ for Γ_i and Γ_c.

Let $\dot{V} = \dot{V}_1 + \dot{V}_2$. Using the bounds for Γ_i and Γ_c, we rewrite $\dot{V}$ as

$$\dot{V} \leq -\sum_{i=1}^N \xi_i^T K_i \xi_i - \frac{k_c}{2\mu_c} \tilde{\theta}_c^T \lambda_{min}\{\mathscr{G}_c\} \tilde{\theta}_c - \frac{\beta_c}{2\mu_c} \tilde{\theta}_c^T \overline{\Gamma_c} \tilde{\theta}_c \\ - \sum_{i=1}^N \frac{k_i}{2\mu_i} \tilde{\theta}_i^T \lambda_{min}\{\mathscr{G}_i\} \tilde{\theta}_i - \sum_{i=1}^N \frac{\beta_i}{2\mu_i} \tilde{\theta}_i^T \overline{\Gamma_i} \tilde{\theta}_i \leq 0. \tag{35}$$

During the time interval $[0, T^*)$, $\dot{V} \leq -\sum_{i=1}^N \xi_i^T K_i \xi_i \leq 0$, which is negative semi-definite. It follows from [25, Theorem 4.8] that ξ_i, $\tilde{\theta}_i$, $\tilde{\theta}_c$, $\tilde{z}_i$ and ξ_c are uniformly bounded and the desired equilibrium $\mathcal{E}^*$ is uniformly stable. Given the states are bounded, we further conclude that $\mathscr{G}_i$ and $\mathscr{G}_c$ are bounded. However, parameter convergence may not be achieved due to the lack of PE.

During the time interval $[T^*, \infty)$, since $\mathscr{G}_i$ and $\mathscr{G}_c$ are full rank, we apply the Barbalat's Lemma [25, Theorem 8.4] and conclude that $\xi_i \to 0$, $\tilde{\theta}_i \to 0$ and $\tilde{\theta}_c \to 0$ as $t \to \infty$, which further implies that $\dot{x}_i \to v^d$ and $\hat{\theta}_i \to \theta_i$ and $\hat{\theta}_c \to \theta_c$.

We next prove $\dot{\xi}_i \to 0$ using [26, Lemma 1]. From (26), the time derivative of $\dot{\xi}_i$, whenever it exists, is given by

$$m_i \ddot{\xi}_i = -\left(K_i + 2[\mathrm{diag}(\tilde{\theta}_i)] \|\xi_i + v^d\| \right) \frac{1}{m_i} \Bigg(-K_i \xi_i \\ + \frac{1}{N} Y_f \hat{\theta}_c - f_i - \mathrm{diag}(\tilde{\theta}_i) \|\xi_i + v^d\| (\xi_i + v^d) \Bigg) \\ + \frac{1}{N} Y_f \dot{\hat{\theta}}_c - \dot{f}_i - \mathrm{diag}(\dot{\tilde{\theta}}_i) \|\xi_i + v^d\| (\xi_i + v^d). \tag{36}$$

From (36), we note that $\ddot{\xi}_i$ is bounded. The points of non-differentiability of $\dot{\xi}_i$ coincide with the points of discontinuity of $G_{k,i}$. By introducing a dwell time in the singular value maximization algorithm, it can be easily ensured that the points of discontinuity of $G_{k,i}$ do not accumulate. Direct application of [26, Lemma 1] then leads to $\dot{\xi}_i \to 0$ as $t \to \infty$.

Since $\dot{\xi}_i \to 0$, (26) indicates that $f_i \to \frac{1}{N} Y_f \hat{\theta}_c$ which further implies that $\sum_{i=1}^N f_i \to Y_f \theta_c = M_c g e_3 + F_d$. Therefore, f_i and z_i converge to f_i^d and z_i^d, respectively. We can similarly use [26, Lemma 1] to prove that $\dot{z}_i \to 0$. It then follows from (30) that $\xi_c \to 0$ which leads to $\dot{x}_c \to v^d$. ■

Remark 1: Without the strict convexity assumption of $P(z_i)$, if $\frac{\partial^2 P(z_i)}{\partial z_i^2}\Big|_{z_i=z_i^d} > 0$, we can write

$$P(z_i) \approx P(z_i^d) + (f_i^d)^T (z_i - z_i^d) + (z_i - z_i^d)^T \frac{\partial^2 P(z_i)}{\partial z_i^2}\Big|_{z_i=z_i^d} (z_i - z_i^d), \tag{37}$$

which means that $P_i(z_i) - P_i(z_i^d) - (f_i^d)^T (z_i - z_i^d)$ is locally positive definite. Then the result in Theorem 1 holds locally. ■

V. Numerical Simulations

We present a simulation of two quadcopters transporting a load. We choose $m_i = 0.75$ kg, $M_c = 1.5$ kg and $r_i = 15$ cm. We used $f_i = kz_i$, where $k = 2.5 \times 10^4$ N/m. We set $r_1 = [0, 0.15, 0]^T$ and $r_2 = [0, -0.15, 0]^T$. We set $v^d(t) = (1 - e^{-t}) \begin{bmatrix} 5.0 & 2.0 & 0 \end{bmatrix}^T$, $k_i = k_c = 0.0033$, $\beta_i = \beta_c = 1$, $\mu_i = \mu_c = 10^{-6}$, and $K_i = \mathrm{diag}(150, 150, 150)$.

The actual drag force coefficients for the quadcopters and the disturbance for the payload are given by $C_i = \begin{bmatrix} 0.2061, 0.2061, 0.2061 \end{bmatrix}^T$ and $F_d = \begin{bmatrix} 6.7315 & 2.6926 & 0 \end{bmatrix}^T$N, respectively. Note that we have chosen $F_d = C_c \|v^d\| v^d$, where C_c is given in [27].

The gains were selected based on the values used in [11] and [12], and were further tuned using trial and error. For example, increasing K_i reduces initial transient response. The gains μ_i and μ_c are kept small to compensate for potentially large Γ_i or Γ_c values to facilitate better convergence of the signals. Note that due to the very small scaling factors μ_i and μ_c in the stabilizing control, the learning estimates do not vary significantly during the first one second.

a) Implementation on a quadcopter: Once F_i is designed from (12), we can compute the desired thrust T_i^{des} and desired attitude angles θ_i^{des}, ϕ_i^{des}, ψ_i^{des} required for each quadcopter from equations (17-21) in [11] and low-level attitude and thrust tracking controllers (e.g., a PD controller) can be implemented to track these desire commands for the ith quadcopter.

As shown below in the simulation results, the CL algorithm uses the first second of time to collect data. During that time period, the CL update is turned off. As discussed in the proof of Theorem 1, the system remains stable. Once the CL update is turned on, the parameters quickly converge.

Fig. 2 shows that the velocities of the agents and the payload converge to v^d. Fig. 3 shows the convergence of the drag coefficients for quadcopter 1. Fig. 4 demonstrates that all the estimation errors for the drag forces and the mass of the payload converge to zero. We observe from Fig. 5 that the contact forces converge to the desired set points. Due to the desired motion being strictly in the $x-y$ plane, the z-component of the velocity only has small nonzero values during the transient response. As a result, the parameters converge slower in the z direction.

b) Simulation Example with a time-varying desired velocity: We examine the performance of the controller for a time-varying $v^d(t)$ defined as

$$v^d(t) = \begin{cases} [t,\ 0.4t,\ 0]^T, & \text{t} \le 5 \text{ s}, \\ [5,\ 2,\ 0]^T, & 5 < \text{t} \le 10 \text{ s}, \\ [-t+15,\ -0.4t+6,\ 0]^T, & 10 < t \le 15 \text{ s}, \\ [0,\ 0,\ 0]^T, & \text{t} \ge 15 \text{ s}. \end{cases} \tag{38}$$

Fig. 6 shows that the velocities of the agents and the payload converge to v^d. Fig. 7 demonstrates that all the estimation errors for the drag forces and the mass of the payload converge to zero. Shown in Fig. 8, the $\tilde{f}_i^d$'s have small steady state errors when $v^d(t)$ is time-varying.

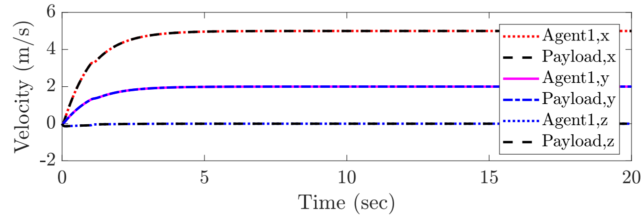

Fig. 2: Linear velocities for both quadcopter 1 and the payload. The x and y components of the velocity converges to 5.0 and 2.0 m/s respectively and the z component converge to zero. Quadcopter 2 has similar velocity convergence.

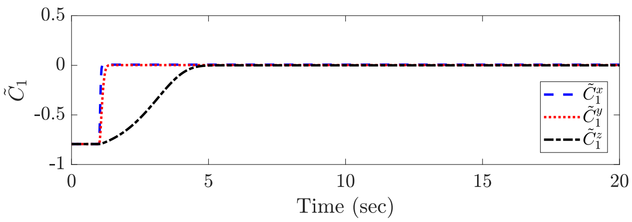

Fig. 3: Estimation errors for drag force coefficient for quadcopter 1 in all 3 directions. Quadrotor 2 has similar convergence.

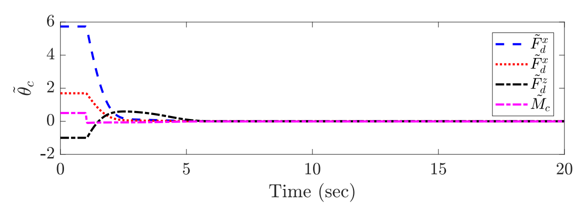

Fig. 4: Estimation errors for drag forces acting on the payload and the mass of the payload.

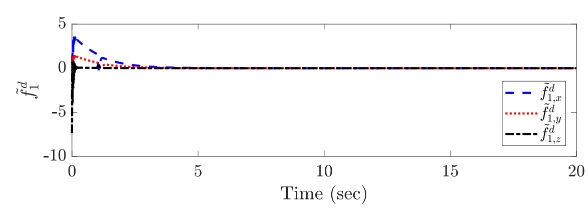

Fig. 5: Contact force error acting on the payload for quadcopter 1.

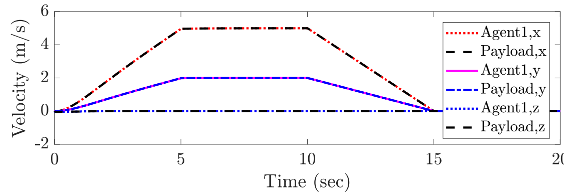

Fig. 6: Linear velocities for both quadcopter 1 and the payload for the time-varying $v^d(t)$ in (38). The controller is able to track $v^d(t)$.

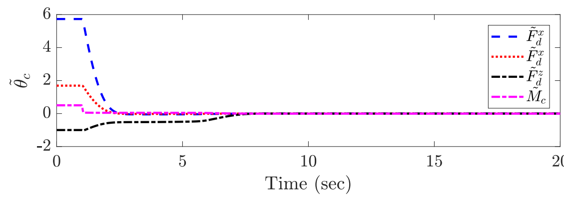

Fig. 7: Estimation errors for drag forces acting on the payload and the mass of the payload for $v^d(t)$ defined in (38).

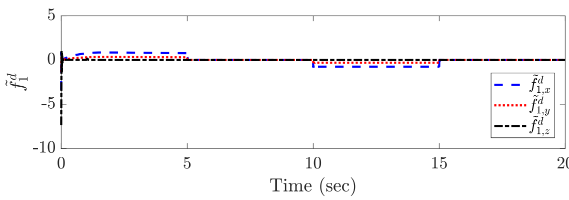

Fig. 8: Contact force error acting on the payload for quadcopter 1 for $v^d(t)$ defined in (38).

VI. Conclusion and Future Work

In this paper, we address the problem of cooperative manipulation of a payload with an unknown mass in the presence of unknown drag forces. We develop a CL based adaptive controller and analyze its stability and convergence properties. We show that the controller guarantees parameter convergence, velocity convergence of the payload and the agents, and contact force regulation. We validate the performance of the controller using two simulation examples. Future work will involve experimental validation of the control law designed in this paper with a group of robots. We also plan to design force control laws coupled with state estimation and address time-varying drag forces on the payload.

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